## Supergravity description of field theories on curved manifolds and a no go theorem

#### Juan Maldacena and Carlos Nuñez

Jefferson Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

#### Abstract

In the first part of this paper we find supergravity solutions corresponding to branes on worldvolumes of the form  $R^d \times \Sigma$  where  $\Sigma$  is a Riemann surface. These theories arise when we wrap branes on holomorphic Riemann surfaces inside K3 or CY manifolds. In some cases the theory at low energies is a conformal field theory with two less dimensions. We find some non-singular supersymmetric compactifications of M-theory down to  $AdS_5$ . We also propose a criterion for permissible singularities in supergravity solutions.

In the second part of this paper, which can be read independently of the first, we show that there are no non-singular Randall-Sundrum or de-Sitter compactifications for large class of gravity theories.

### 1 Introduction

In the first part of this paper we study the large N limit of branes wrapped on nontrivial cycles. More precisely we consider a d+2 dimensional field theory defined on a space of the form  $R^d \times \Sigma_g$  where  $\Sigma_g$  is a genus g Riemann surface. These theories reduce to d dimensional field theories at low energies, at energies small compared to the inverse size of the Riemann surface. We consider supergravity solutions that describe the flow between the d+2 dimensional field theory and the d dimensional field theory. Our d+2 dimensional field theories are superconformal and have the maximum amount of supersymmetry. More precisely we consider  $\mathcal{N}=4$  super-Yang-Mills in four dimensions and the (0,2) theory in six dimensions. These will give rise to two and four dimensional field theories respectively. These field theories will have less supersymmetry. We consider situations where the supersymmetry gets reduced to 1/2 or 1/4 of the original supersymmetry of the d+2 dimensional theory. The amount of supersymmetry preserved depends on how the two dimensional surface  $\Sigma_g$  is embedded in a higher dimensional space. In the field theory limit, these different embedding possibilities translate into different normal bundles and therefore different external SO(n)gauge fields on the worldvolume theory, where n is the number of directions normal to the brane. There are several cases where the resulting d dimensional field theory is conformal. In those cases we find an  $AdS_{d+1}$  geometry in the IR. These geometries in the IR have an interest of its own and provide new AdS/CFT examples. In particular, we find new  $\mathcal{N}=2$  and  $\mathcal{N}=1$  superconformal field theories in four dimensions that arise from wrapping M5 branes on negatively curved Riemann surfaces (surfaces with q > 1). They are dual to  $AdS_5$  warped compactifications of eleven dimensional supergravity. These are  $AdS_5$  fibrations over a six dimensional space, the five dimensional warp factor depends on the six dimensional coordinates<sup>1</sup>. Similarly we find some compactifications of IIB string theory to  $AdS_3$  that preserve (2,2) supersymmetry.

The basic technique that we use to find the solutions is the following. The field theories on branes are twisted theories [4]. What this means is that together with the coupling to the curvature of the brane worldvolume there is a coupling to an external SO(n) gauge field, if n is the number of transverse directions to the brane. So we have a field theory on a curved space coupled to a SO(n) gauge field. In supergravity this translates into boundary conditions at the boundary of  $AdS_{d+3}$  for the metric and the SO(n) gauge fields. Fortunately these are modes of gauged supergravity, so that we can consider just the d+3 dimensional gauged supergravity equations. We further

<sup>&</sup>lt;sup>1</sup>Related compactifications were found in [1, 2], but in their case they the warp factor was going to zero at some points in the internal space. In our case the metric is completely smooth. In [3] similar configurations were studied and the conditions for preserved supersymmetry were studied in great generality and some solutions were found corresponding to intersecting branes with a certain amount of "smearing" in one of the transverse dimensions.

consider only constant curvature Riemann surfaces and simple embedding of the spin connection into the SO(n) connection. A more general analysis is left for the future.

Some of these supergravity solutions can be viewed as compactifications of d+3 dimensional gauged supergravities on  $AdS_{d+1} \times \Sigma_{g>1}$  with magnetic fluxes on  $\Sigma$ . They are similar in spirit to the solutions in [5, 6, 7, 8, 9].

Since we have obtained a large family of  $AdS_5$  compactifications it is natural to ask if we could find a smooth Randall-Sundrum [10] compactification based on these. In section 6 we show that it is not possible to find a smooth Randall-Sundrum compactification of usual supergravity theories. In fact we prove something even more general, it is not possible to find warped compactifications which have only singularities where the warp factor is non-increasing as we approach the singularity. We also show that there are no deSitter compactifications. This section is self-contained and can be read independently of the rest of this paper. For other no go theorems for supersymmetric RS compactifications of 5d gauged supergravity see [11, 12, 13].

There are many papers in the literature which consider compactifications similar to the ones considered here [14, 6, 7, 8, 9, 15, 16, 17, 18, 19, 20, 21]. While this paper was in preparation the paper [22] appeared which has some overlap with ours and presents another interesting method, based on [3], for obtaining a large class of solutions corresponding to branes on Riemann surfaces.

This paper is organized as follows. We first describe the general idea behind twisted field theories and how we plan to find the gravity solutions. In section 3 we consider  $\mathcal{N}=4$  super Yang Mills on  $R^2\times\Sigma$ . We consider two possible twisting which preserve (4,4) and (2,2) supersymmetry in 1+1 dimensions. In section four we consider the (0,2) superconformal field theory that lives on M5 branes compactified on spaces of the form  $R^4\times\Sigma$ . We consider again two cases, preserving  $\mathcal{N}=2,1$  supersymmetry in four dimensions. In section 5 we discuss a criterion for allowed singularities in gravity theories that are dual to field theories. In section 6 we discuss the absence of certain warped compactifications or de Sitter solutions in a class of gravity theories, including 11d supergravity, IIB, IIA and massive IIA.

In the appendix we give some more details on our calculations.

## 2 General Idea

If we start with a supersymmetric field theory and we put it on a curved manifold  $\Omega$  then, in general, we will break supersymmetry since we will not have a covariantly constant spinor, obeying  $(\partial_{\mu} + \omega_{\mu})\epsilon = 0$ . If the supersymmetric theory has a global R-symmetry, then we can couple the theory to an external gauge field that couples to the R-symmetry current. If we choose the external gauge field to be equal to the spin

connection  $A_{\mu} = \omega_{\mu}$  (the precise meaning of this equation will be explained below) then we see that we can find a covariantly constant spinor since  $\partial_{\mu} + \omega_{\mu} - A_{\mu}\epsilon = \partial_{\mu}\epsilon$ , so that we can just consider a constant spinor. The resulting theory is a so called "twisted" theory, since we can view the the coupling to the external gauge field as effectively changing the spins of all fields. The supersymmetry parameter becomes a scalar. Though it might sound like a contrived way of preserving supersymmetry, it is precisely the way that branes wrapping on non-trivial cycles in M-theory or string theory compactifications manage to preserve some supersymmetries [4]. In this case  $\Omega$  is the worldvolume geometry of the cycle and the external gauge field takes into account the fact that the directions normal to the cycle form a non-trivial bundle, the normal bundle, and  $A_{\mu}$  is the connection on this normal bundle. The condition that the cycle preserves some supersymmetry then boils down to the condition that the spin connection is equal to the gauge connection. So if we are interested in understanding field theories arising on branes wrapping non-trivial cycles, then we will have to study these twisted theories. Let us first clarify the nature of the limit in which we decouple this twisted field theory from the full original string theory. We consider a brane wrapping a cycle and we take the decoupling limit  $l_p \to 0$  as in [23] keeping the volume of the cycle fixed. In this limit we get a field theory on the brane that is twisted as above, due to the non-trivial embedding of the cycle in the ambient space. If the theory on the brane is conformal before we wrap it, this is all we have to do. If we have a D-p-brane with  $p \neq 3$  then we also should scale the string coupling as in the flat case [24]. Notice that in this limit the theory is not sensitive to the global geometry of the spacetime where the cycle is embedded. The reason is that a finite fluctuation of the scalar field parameterizing transverse displacements corresponds to an infinitesimal fluctuation in the position of the brane. In other words, in the scaling limit we are considering, the size of the Calabi Yau (or any other space where the brane is embedded) is fixed, while the typical fluctuation of the position of the brane goes to zero as  $l_p$  goes to zero. This of course does not imply that the scalars associated to motions of the brane will have definite expectation values. Whether they do or not will depend on the number of non-compact dimensions of the theory. We will discuss this more later. Let us consider some examples. Suppose we have type IIB string theory on  $R^6 \times K3$ . We can wrap a D3 brane on an  $S^2$  inside the K3 manifold leaving two non-compact directions. The worldvolume is then  $\Omega = \mathbb{R}^2 \times \mathbb{S}^2$ . In this case, the spin connection is in a U(1) subgroup of the tangent group SO(3,1), since the curvature is purely in the  $S^2$  directions. The brane has 6 normal directions. Two of them are in the directions of K3 that are normal to the  $S^2$ . They will form a non-trivial U(1) normal bundle. It turns out that for holomorphically embedded spheres, the spin connection is essentially equal to the connection on the U(1) normal bundle. The other four normal directions are totally flat, with no gauge field. This gauge connection will

break the SO(6) R-symmetry group into  $U(1) \times SO(4)$ . Out of the 16 spinors that generate the supersymmetries of  $\mathcal{N}=4$  Yang Mills, there will be only half for which the chirality on  $S^2$  and the U(1) charges are correlated so that the spin connection and gauge connection cancel. In terms of the two dimensional low energy theory on  $R^2$  the theory has (4,4) supersymmetry. These kind of theories were considered in [25, 26]. Below we find the supergravity solutions associated to these twisted field theories. For this particular example we find that there is a family of supergravity solutions and that they all seem to have a singularity in the IR. We argue later that this singularity is associated to the IR properties of the brane theory and that only the singularities of an allowable type, according to the criterion in section 5 or the one in [27], produce the right physics. We will consider however some other examples, involving branes wrapping negatively curved Riemann surfaces, for which there are non-singular solutions which in the IR have an AdS form. For example if we wrap a D3 brane on a genus q > 1 Riemann surface times  $R^2$  with a particular normal bundle we specify below then the supergravity solution interpolates between an  $AdS_5$  region close to the boundary and an  $AdS_3$  region in the IR, corresponding to the fact that the theory in the UV is just 3+1 dimensional SYM and it flows to a 1+1 dimensional conformal field theory. So these are examples of flows "across dimensions", the four dimensional conformal field theory flows to a two dimensional conformal field theory.

We only consider cases where the genus of the Riemann surface  $\Sigma_g$  is  $g \neq 1$ , since in the case of g = 1 the constant curvature metric is just the metric on a flat  $T^2$  and the theory is identical to the untwisted theory.

We will find similar examples for M5 branes wrapping on negatively curved Riemann surfaces. These will give us a new family of examples of four dimensional conformal field theories and their associated  $AdS_5$  compactifications of M-theory. The case analyzed in [1, 28] would probably arise as a singular limit of the smooth solutions analyzed here. In principle it should also be possible to find the solutions in our paper using the methods of [3, 22] but we will not pursue that here. In Horava-Witten M-theory compactifications to 3+1 dimensions we could have M5 branes wrapping on Riemann surfaces in the CY manifold at some points in moduli space. Our analysis implies that coincident branes will typically give rise to conformal field theories. More accurately, that they give rise to conformal field theories in the large N limit and the large CY volume limit. Finite N effects, or finite volume effects of the CY manifold could destroy conformality. Since having a conformal field theory, or a theory with a logarithmically running coupling would give a natural explanation of the gauge hierarchy problem, it is interesting that these arise quite naturally in these compactifications. One should also be careful with this conclusion because in Horava-Witten compactifications we will also generically have some fluxes of the four form field strength. These might induce relevant perturbations of the field theory, as in [29].

Since we are dealing with the theory of coincident M5 branes we find it hard to give a purely four dimensional field theory description of the theory in the IR. It looks like it should be possible to say what it is more precisely, but leave this to the future. Dimensional reducing the M5 solutions we get D4 solutions corresponding to D4 branes wrapped on Riemann surfaces. In this case we can state more precisely what the 3 dimensional field theories are.

In this paper we will consider Riemann surfaces with constant curvature. We think of them as  $H_2/\Gamma$  where  $\Gamma$  is a discrete subgroup of SL(2,R). Notice that for our purposes, which is to find a precise sugra solution, the precise metric on the Riemann surface does matter. One can find solutions for non-constant curvatures by the methods of this paper or by the methods of [22] where a large family of solutions was found.

## 3 Twisted 4d N=4 SYM

The possible twistings of N=4 SYM were considered in [25]. These twistings are distinguished by different ways of embedding the spin connection in the SU(4) R-symmetry group. In principle we could consider the most general case by the techniques of this paper. But in order to keep formulas simple we will concentrate on four dimensional spaces of the form  $R^2 \times \Sigma_2$  where  $\Sigma_2$  is a two dimensional manifold. In this case the spin connection is in a U(1) subgroup. So, different twistings of the theory are defined by different embedding of this U(1) in SU(4) = SO(6). We will consider two cases, one which preserves (4,4) supersymmetry and one preserving (2,2) supersymmetry.

Throughout this section we work in units where the five dimensional  $AdS_5$  radius is one. In order to restore the radius dependence of the solutions we just multiply the five or ten dimensional metrics we will write by  $R_{AdS_5}^2 = \sqrt{4\pi g_s N} \alpha'$ .

## 3.1 Twists preserving (4,4) susy

The first twisting that we consider corresponds to picking a U(1) in SO(6) in such a way that we break SO(6) oup SO(2) imes SO(4), where the first SO(2) is the U(1) that we are picking inside SO(6). In other words, if we think of SO(6) as acting on six coordinates  $\phi^I$ , then the U(1) is the group of rotations in the 12 plane. This twisting turns out to be exactly the same as the one considered in  $[25]^2$  and the resulting field theories were analyzed in [26]. In order to explain more clearly how this works, let us consider a field  $\phi$  in the Yang-Mills Lagrangian, with spin s under the SO(2) spin connection on  $\Sigma_2$ 

<sup>&</sup>lt;sup>2</sup>In [25] more general manifolds were considered, but it was noted that if the four dimensional manifold is Kahler (as in our case), then only the U(1) mentioned above has a non-trivial gauge connection

and U(1) charge q. The Lagrangian is obtained from the flat Lagrangian, by replacing ordinary derivatives of  $\phi$  in the  $\Sigma$  directions by covariant derivatives

$$\mathcal{D}_{\mu}\phi = (\partial_{\mu} + si\omega_{\mu} + iqA_{\mu})\phi \tag{1}$$

where  $\omega_{\mu} = \epsilon_{ab}\omega_{\mu}^{ab}/2$  with  $\omega_{\mu}^{ab}$  the usual spin connection. If the metric on  $\Sigma_2$  is  $ds^2 =$  $e^{2h}(dx^2+dy^2)$  then the spin connection is  $\omega_{\mu}=\epsilon_{\mu\nu}\partial_{\nu}h$ , and we also have  $A_{\mu}=\omega_{\mu}$ . This implies that spinors with s = -q can be covariantly constant. In fact they are actually constant, the twisting effectively made them scalars. In this situation the  $SO(1,3) \times SO(6)$  symmetry group of the tangent and normal bundles is decomposed as  $SO(1,1) \times SO(2)_{\Sigma} \times U(1) \times SU(2)_L \times SU(2)_R$  and the preserved spinors transform in the  $(+, \pm, \mp, 1, 2)$  and  $(-, \pm, \mp, 2, 1)$  representations. So we have (4,4) supersymmetry, in 1+1 dimensional notation. Let us consider some examples where these theories arise. Suppose we have a compactification of the form  $R^6 \times K3$ . Then we can wrap a D3 on a holomorphic Riemann surface inside K3 and we obtain a field theory on the D3 which is of the above from. The  $SO(4) = SU(2)_L \times SU(2)_R$  symmetry of the field theory is the rotational symmetry in the four directions in  $R^6$  that are orthogonal to the brane. More precisely, the limit in which we get the above field theory is a limit where we take  $\alpha' \to 0$  keeping the size of the Riemann surface (and the K3) fixed. Therefore, this limit corresponds to a large volume compactification, large in string units. In this limit the directions normal to the Riemann surface become effectively non-compact, since the brane explores only an infinitesimal neighborhood of the surface. In other words, a field  $\phi$  parameterizing fluctuations of the position of the brane is related to the actual displacement by  $r = \alpha' \phi$  so that the actual displacement goes to zero as  $\alpha' \to 0$ . We can then take a further low energy limit where we consider energies much smaller than the size of the Riemann surface. In this limit we obtain a two dimensional effective theory, which in the IR is a (4.4) superconformal theory [26]. With the order of limits that we took, this conformal field theory has a non-compact target space since we are only exploring a neighborhood of the Riemann surface. The fact that the target space is non-compact implies that this CFT does not have a well defined vacuum state. We would have obtained a different theory if we had taken the size of the Riemann surface and K3 fixed in string units. In that case the target space would have been essentially compact<sup>3</sup>, and quantum fluctuations would have explored the whole moduli space of Riemann surfaces. The resulting theory would be (a T-dual version of) the familiar D1 - D5 system, if the genus of the Riemann surface is q > 1.

Let us describe in more detail about the Lagrangian of these theories. The full Lagrangian can be found in [25]. Here we give only some terms that are relevant in comparing with gravity solutions. The only parts of the Lagrangian that are different from the usual  $\mathcal{N}=4$  Lagrangian are the terms involving covariant derivatives along  $\Sigma$ 

<sup>&</sup>lt;sup>3</sup>It still might still have some non-compact directions via effects such as the ones discussed in [30].

or fields that are charged under the U(1) part of the normal bundle that has a non-zero gauge connection. Let us consider the part of the Lagrangian involving the two twisted scalar fields. These are the two scalar fields parameterizing fluctuations of the surface in the two normal directions which transform under U(1). Let us arrange these two fields into a complex field  $\mathcal{Z} = X^1 + iX^2$ . The quadratic terms in the Lagrangian involving these fields are

$$S = \int Tr\{|\mathcal{D}_z \mathcal{Z}|^2 + |\mathcal{D}_{\bar{z}} \mathcal{Z}|^2 + \frac{1}{4}R|\mathcal{Z}|^2\}$$
 (2)

The first two terms are the terms we would obviously expect from (1) and the last term is a curvature coupling that was derived from supersymmetry in [25], but in the case of single D-brane it can also be obtained directly by expanding the Nambu action for a brane on this surface. The other scalar fields  $\phi^I$ , I=1,...,4 have a simple Lagrangian of the form  $(\partial \phi^I)^2$ . Note that integrating by parts the first term in (2) we can recast it as the second term up to a commutator  $[\mathcal{D}_z, \mathcal{D}_{\bar{z}}]$ . This commutator precisely cancels the curvature term in (2). We can choose a holomorphic basis for the normal bundle so that  $\mathcal{D}_{\bar{z}}\mathcal{Z} = \partial_{\bar{z}}\mathcal{Z}$ . This implies that holomorphic sections of the normal bundle  $\mathcal{Z}(z)$  are solutions of the equations of motion and describe configurations with the same energy as the original configuration. This is precisely what we expect since any holomorphic deformation of the surface preserves supersymmetry. Of course, whether these deformations exist globally on the surface or not depends on the global aspects of the geometry and the normal bundle.

Let us look for the supergravity dual of these field theories. Since these are just  $\mathcal{N}=4$  3+1 dimensional SYM theories on some particular backgrounds, we expect to be able to find the gravity dual by starting with  $AdS_5 \times S^5$  and changing the asymptotic boundary conditions to reflect the fact that the theory is defined on  $R^2 \times \Sigma$  and is coupled to an SO(6) gauge field. This is easy to achieve. We impose that at the boundary of  $AdS_5$  the metric behaves as

$$ds^2 \sim \frac{-dt^2 + dz^2 + e^{2h}(dx^2 + dy^2) + dr^2}{r^2}$$
 for small  $r$  (3)

where  $ds_{\Sigma}^2 = e^{2h(x,y)}(dx^2 + dy^2)$  is the metric of the two dimensional surface. Similarly we impose that the  $AdS_5$  SO(6) gauge fields are asymptotic to the corresponding field theory values. This translates into a condition on components of the metric with one index in  $AdS_5$  and one index on  $S^5$ . In other words we require  $g_{\phi\alpha} \sim A_{\alpha} \sim \epsilon_{\alpha}^{\ \beta} \partial_{\beta} h$  near the boundary. These two conditions are rather obvious. A bit less obvious is the fact that we also need to turn on an operator in the **20** of SO(6). This becomes apparent once we look at the curvature coupling in (2) and realize that that coupling is not present for the other four scalar fields. This means that the operator

$$\mathcal{O}_2 = Tr\left[\frac{2}{3}|\mathcal{Z}|^2 - \frac{1}{3}(\phi^{1^2} + \dots + \phi^{4^2})\right]$$
 (4)

is turned on. We also have the singlet operator turned on, with the expected coefficient proportional to the scalar curvature [31]. So we can rewrite the curvature coupling in (2) as

$$S = \frac{1}{2} \int R\left(\frac{1}{6}(|\mathcal{Z}|^2 + \phi^{1^2} + \dots + \phi^{4^2}) + \frac{1}{2}\mathcal{O}_2\right)$$
 (5)

Fortunately all the operators that are turned on correspond to fields in the five dimensional gauged supergravity multiplet. This is a general feature for these twisted theories, even in the most general curved backgrounds. In order to find the gravity solutions we can therefore consider just the five dimensional gauged supergravity equations. If we were interested in the most general twisted theory we would have to use the full  $\mathcal{N}=8$  gauged supergravity of [32]. However, in our case the connection is in U(1) so we can further use a U(1) truncation of the equations of the form considered in [33]. Furthermore, in [33] one can find formulas to express the ten dimensional solution given any solution of the truncated five dimensional equations. We will consider therefore a theory involving the five dimensional metric, a U(1) gauge field and a scalar field  $\varphi$  which is dual to the operator  $\mathcal{O}_2$  appearing above. In order to find supersymmetric solutions we look at the supersymmetry variation equations of the fermionic fields. These can be read of from [34, 6] as explained in the appendix.

$$\frac{1}{\sqrt{6}}\delta\lambda = -\frac{1}{24}e^{-2\varphi}\Gamma^{\mu\nu}F_{\mu\nu}\epsilon - \frac{i}{4}\Gamma^{\mu}\partial_{\mu}\varphi\epsilon + \frac{i}{6}(e^{2\varphi} - e^{-\varphi})\epsilon$$

$$\delta\psi_{\mu} = \mathcal{D}_{\mu}(\omega)\epsilon + \frac{i}{24}e^{-2\varphi}(\Gamma^{\nu\rho}_{\mu} - 4\delta^{\nu}_{\mu}\Gamma^{\rho})F_{\nu\rho}\epsilon + \frac{1}{6}\Gamma_{\mu}(2e^{-\varphi} + e^{2\varphi})\epsilon - \frac{i}{2}A_{\mu}\epsilon$$
(6)

In order to solve this equation we make the following ansatz for the metric

$$ds^{2} = e^{2f}(dr^{2} + dz^{2} - dt^{2}) + \frac{e^{2g}}{y^{2}}(dx^{2} + dy^{2})$$
(7)

where f and g are functions of r to be determined. For simplicity we are considering constant curvature Riemann surfaces of genus g > 1<sup>4</sup>. When  $r \to 0$  the boundary conditions are  $f(r) \sim g(r) \sim -log(r)$ . The gauge field is  $A_x = 1/y$ . This value comes from demanding equality with the spin connection. Setting to zero the supersymmetry variations we obtain, see the appendix,

$$g' = -e^{f} \left[ \frac{1}{3} (2e^{-\varphi} + e^{2\varphi}) - \frac{1}{3} e^{-2g - 2\varphi} \right]$$

$$f' = -\frac{1}{6} e^{f} \left[ 2(2e^{-\varphi} + e^{2\varphi}) + e^{-2g - 2\varphi} \right]$$

$$\varphi' = \frac{1}{3} e^{f} \left[ 2(-e^{-\varphi} + e^{2\varphi}) + e^{-2g - 2\varphi} \right]$$
(8)

<sup>&</sup>lt;sup>4</sup>The reader should not be confused by the fact that g denotes both the function g(r) appearing in (7) and the genus.

The solution is given by

$$e^{-3\varphi} = 1 + e^{-2\rho} \left[ \frac{1}{2} \log(e^{2\rho} - \frac{1}{2}) + C_1 \right]$$

$$e^{2g} = e^{2\rho} e^{-\varphi}$$

$$e^{2f} = \left( e^{2\rho} - \frac{1}{2} \right) e^{-\varphi}$$

$$\left( \frac{dr}{d\rho} \right)^2 e^{2f} = \frac{e^{4\rho}}{(e^{2\rho} - \frac{1}{2})^2} e^{2\varphi}$$
(9)

Notice that using the last line we can rewrite the metric in terms of the new radial coordinate  $\rho$ ,  $\rho \to +\infty$  corresponds to the boundary.  $C_1$  is an integration constant, there is of course another trivial integration constant which amounts to shifting f by a constant. Note that once we solve the equations for  $H_2$  we can quotient the solution by a subgroup  $\Gamma$  of SL(2,R) that produces the Riemann surface  $\Sigma_g = H^2/\Gamma$ . This group also acts in the U(1) that we are twisting. That implies that the spinor parameter that generates the preserved supersymmetry transforms like a scalar under SL(2,R) transformations and it therefore survives the quotienting procedure.

We see that there is a singularity of the metric at  $\rho = \rho^*$  where  $e^{-3\varphi(\rho_*)} = 0$ . This singularity is qualitatively the same regardless of the value of  $C_1$ . These singularities are related to regions in the Coulomb or Higgs branches of the theory. Let us explore this a bit more. If we expand the first term in equation (9) for large  $\rho$  we find that  $\varphi \sim -e^{-2\rho}\rho - e^{-2\rho}C_1$ . The logarithmic term is related to the fact that we are inserting the operator  $\mathcal{O}_2$  as in (5) the subleading term can be thought of as the expectation value of this operator [35]. We would like to relate the sign of  $C_1$  to the sign of the expectation value of  $\mathcal{O}_2$ . We can do that by looking at [36] where configurations in  $\mathcal{N}=4$  super Yang-Mills with both expectation values for  $\mathcal{O}_2$  were considered. We find from their paper that  $C_1 \sim \langle \mathcal{O}_2 \rangle$ . From the explicit expression for the operator  $\mathcal{O}_2$  (4) we see that configurations with  $\langle \mathcal{O}_2 \rangle > 0$  correspond to Higgs branch configurations while ones where  $\langle \mathcal{O}_2 \rangle < 0$  correspond to the Coulomb branch. Since we also have the operator inserted, it is a bit hard to separate its expectation value, so we should trust this criterion only for  $|C_1|$  large. The behavior at the singularity is very similar for both signs of  $C_1$ . These singularities look very much like the singularities in the Coulomb branch, where branes are distributed on an  $S^3$  as in [36], even though we expect that negative values of  $C_1$  should correspond to the Higgs branch. This seems to be related to the Higgs-Coulomb correspondence as in [37, 38, 39] where it was shown that some singularities in the Higgs branch look very similar to the near region of the Coulomb branch. What seems surprising, when we compare this system to the familiar D1-D5 system, is that we do not find any solution that has an  $AdS_3$  region in the IR. We think that this is related to the fact that the (4,4) superconformal field theory that we are dealing with here has a non-compact target space. This non-compactness is different in

nature from the one that arises at special points in moduli space for the D1-D5 system [30]. In the D1-D5 system the singularities are small instanton singularities which require tuning of order  $Q_5$  parameters out of the order  $Q_1Q_5$  parameters in instanton moduli in order to reach them. In our (4,4) we have another kind of non-compactness of the moduli space. This non-compactness is related to the possibility of moving the branes in the directions orthogonal to the Riemann surface. In other words, we are saying that the non-compactness of the Higgs branch in this case is very similar to the non-compactness we would have if we were to consider the D1-D5 system but with the internal space replaced by  $R^4$  instead of  $T^4$  and a finite number  $Q_1$  of one branes. It was shown in [40] that that system has no  $AdS_3$  region.

Let us find the theory we expect at low energies from the field theory point of view. It was argued in [4] that this (4,4) superconformal field theory was a conformal field theory whose target space is Hitchin space. Its central charge is  $c = 6N^2(g-1)$ . This moduli space is non-compact and that seems to be preventing the  $AdS_3$  solutions. Another way to say what the low energy theory is; is the following. Suppose we add two more dimensions along the brane so that we have a D5 brane wrapped on  $\Sigma_q$ , g > 1. Then we have an  $\mathcal{N}=2$  theory in four dimensions. This four dimensional theory is a U(N) theory with g adjoint hypers. A way to see this is to go to weak coupling and calculate explicitly the number of low energy modes and their susy properties, see [41] for a related discussion. We can calculate that number using an index argument. Supersymmetry then determines the form of the Lagrangian. This theory has a Higgs branch of real dimension  $4N^2(g-1)$ . We can now dimensionally reduce two of the dimensions of the D5 brane to make it a D3 brane. So in the IR we have a CFT which is a sigma model whose target space is the Higgs branch of the gauge theory. A way of thinking about a point in this Higgs branch was given in [42]. The N coincident branes are moved and their intersections "blown up" in such a way that we have a single surface which now has  $2N^2(g-1)$  geometric moduli and the same number of U(1) worldvolume gauge field Wilson lines. Of course, in the  $\alpha' \to 0$  limit that we are taking this single surface has a thickness smaller than  $\alpha'$  and only explores a small neighborhood of the original surface but not the global structure of the manifold where the surface  $\Sigma$  is embedded.

The singularities in (9) are allowed under the criterion given in section 5 which demands that  $g_{00}$  does not increase as we approach the singularity or the one given in [27], which demands that the potential for the scalars should be bounded above in the solution. See the appendix for the explicit computation of the potential.

It is also possible to find the solution for the case that the branes are wrapped on  $R^2 \times S^2$ . Since we can go from the metric in  $H_2$ ,  $ds^2 = d\theta^2 + \sinh^2\theta d\psi^2$ , to minus the metric on  $S^2$  by taking  $\theta \to i\theta$  we can find the equations for the branes on  $R^2 \times S^2$  by formally replacing  $e^{2g} \to -e^{2g}$  on the sphere by taking  $\theta \to i\theta$  in (8). Then we get a

solution similar to (9) that reads

$$e^{-3\varphi} = 1 + e^{-2\rho} \left[ -\frac{1}{2} \log(e^{2\rho} + \frac{1}{2}) + C_1 \right]$$

$$e^{2g} = e^{2\rho} e^{-\varphi}$$

$$e^{2f} = \left( e^{2\rho} + \frac{1}{2} \right) e^{-\varphi}$$

$$\left( \frac{dr}{d\rho} \right)^2 e^{2f} = \frac{e^{4\rho}}{(e^{2\rho} + \frac{1}{2})^2} e^{2\varphi}$$
(10)

Again we interpret  $C_1$  as giving the expectation value of the operator  $\mathcal{O}_2$ . In this case something interesting happens. For large enough negative values of  $C_1$  we have a singularity in the IR that looks very similar to that analyzed for the case of  $H^2$ . For positive enough values of  $C_1$ , corresponding to positive values of  $\langle \mathcal{O}_2 \rangle$ , we find that the singularity is not allowed by the criterion in section 5 or Gubser's criterion [27]. This is very fortunate because we do not expect this field theory to have a Higgs branch. This is due to the fact that the scalars normal to the sphere have no zero modes as can be seen from the fact that (2) is positive for non-zero values of  $|\mathcal{Z}|$  if R > 0. The reduction of N = 4 SYM on  $S^2$  was studied in [43]. This theory does have a Coulomb branch and indeed we see that in the supergravity solution.

When we talk about Coulomb and Higgs branches through this section we should remember that vacuum expectation values for massless fields are not well defined in two dimensions. So we should interpret these solutions as describing some semi-classical states were some aspects do not change very quickly with time. Once the singularities get resolved by the full theory, by taking into account the IR degrees of freedom that give rise to the singularities then we expect that the solution would change slowly with time as the vevs of massless fields drift through moduli space.

## 3.2 Twists preserving (2,2) supersymmetry

Another way in which we can embed the U(1) group of the spin connection is the following. Consider the  $SO(2)^3$  subgroup of SO(6) which corresponds to rotations within three orthogonal planes. Let us call these three generators  $T_i$ , i=1,2,3. We can take the generator  $T=\frac{1}{2}T_2+\frac{1}{2}T_3$ . This breaks  $SO(6)\to SO(2)\times SO(2)\times SU(2)$ . Under this choice of subgroup the scalars split into two for which the normal bundle is trivial and four for which the normal bundle is such that they effectively become spinors. Another way to state this choice is to consider the splitting of  $SO(6)\to SO(2)\times SO(4)$  that we considered in the previous section but now we take the gauge connection in  $U(1)\subset SU(2)_L\subset SO(4)$ .

It can be shown that a Riemann surface in a CY manifold typically has a normal bundle which is topologically like the one we are considering here [44]. We expect then

that if we wrap a D3 brane on this Riemann surface we get a theory that is similar to the theories that we are considering here. In these more general theories the normal bundle does not generically have the additional SU(2) symmetry mentioned above.

Let us consider the bosonic Lagrangian for this theory. Let us call  $W_1, W_2$  the two complex fields parameterizing the four directions where the gauge field is nonzero. This Lagrangian will be similar to (2) but the coefficient of the curvature term will be different. In principle we should get these terms by demanding supersymmetry. However we will find them by demanding holomorphicity, as in the discussion below (2). The Lagrangian is then

$$S = \int Tr\{\sum_{i} |\mathcal{D}_{z}\mathcal{W}_{i}|^{2} + |\mathcal{D}_{\bar{z}}\mathcal{W}_{i}|^{2} + \frac{1}{8}R|\mathcal{W}_{i}|^{2}\}$$

$$\tag{11}$$

Again the remaining two untwisted scalars have an action of the form  $\int (\partial \phi)^2$ . The presence of the curvature term in (11) implies, as in the discussion around (5), that we are turning on the operator in the **20**. It turns out to be the same operator that we were turning on before but with a different coefficient.

When we consider the IR limit of this theory, at energies smaller than the size of the Riemann surface again we expect to find a conformal field theory. We can compute the central charge of this theory by an anomaly argument. When we reduce the theory to two dimensions we find left and right moving massless fermions with different transformation properties under the R-symmetry group. Let us classify the massless fermions according to their transformation properties under the SO(2) ×  $SO(2) \times SO(2)$  subgroup of SO(6). These fermions come from the four dimensional gauginos. There are four possibilities for their charges: (A) (+,+,+), (B) (-,-,+), (C) (-,+,-) (D) (+,-,-). These have positive chirality in SO(6), their complex conjugates have negative chirality in SO(6). Now we can calculate the number of fermion zero modes on  $\Sigma_q$  via an index theorem. Let us do the calculation in the weak coupling limit where we have essentially  $N^2$  free fields. These are fermions which are coupled to an external gauge field, which is the gauge field implementing the twisting, whose generator was defined as T above. The charges for the fermions fields are  $t_A = 1/2$ ,  $t_B = -1/2$ ,  $t_C = t_D = 0$ . The index theorem says that the difference between the number of zero modes on  $\Sigma_g$  with positive and negative chirality is

$$n^{+} - n^{-} = \frac{1}{2\pi} t \int_{\Sigma} F = \frac{t}{2\pi} \int_{\Sigma} R = 2t(g - 1) ,$$
 (12)

where we used that the gauge connection is the same as the spin connection. Since the ten-dimensional chiralities of the gauginos are positive, and we have taken them to have positive SO(6) chirality, then we conclude that the chiralities in 1+1 dimensions are the same as those in  $\Sigma$ . So we conclude that the difference between the number of left and right movers  $n_R - n_L$  with given charges is also given by (12). Now it is useful to define  $U(1)_L$  and  $U(1)_R$  symmetries that will become the  $U(1)_L$  and  $U(1)_R$  symmetries of the CFT in the IR. In order to do this we note that the right-moving preserved supercharge has charges as those of fermions in A, while the left-moving one has charges as those in B. So we define  $T_R = \frac{1}{2}T_1 + \frac{1}{2}T_2 + T_3$  and  $T_L = -\frac{1}{2}T_1 - \frac{1}{2}T_2 + T_3$  with this definition we see that the supersymmetry parameter generating the right moving supersymmetries has charge  $(q_R, q_L) = (1, 0)$  and vice versa for the left moving one. These symmetries are anomalous and their anomalies are given by the usual formulas  $k_R = \sum_i (n_R^i - n_L^i)(q_R^i)^2$  where the sum runs over all fermions in the theory. Using (12) we get  $k_L = k_R = N^2(g-1)$ . Since the anomalies should be the same at weak and strong coupling, and since the (2,2) superconformal algebra links the anomaly to the central charge we can compute the central charge in the field theory to be

$$c_L = c_R = 3N^2(g-1) . (13)$$

We can find the supergravity solution by the method we explained above. We start by writing the supersymmetry variation equations, see the appendix,

$$g' = -\frac{1}{3}e^f[(2e^{-\varphi} + e^{2\varphi}) - e^{-2g}e^{\varphi}]$$
 (14)

$$f' = -\frac{e^f}{6} [2(2e^{-\varphi} + e^{2\varphi}) + e^{-2g}e^{\varphi}]$$
 (15)

$$\varphi' = \frac{2}{3}e^f[(-e^{-\varphi} + e^{2\varphi}) - \frac{1}{4}e^{-2g}e^{\varphi}]$$
 (16)

We could not solve these equations completely. However we could partially solve them by finding an analytic expression relating q and  $\varphi$ 

$$e^{2g+\varphi} = e^{2g-2\varphi} + \frac{g+2\varphi}{2} + C_1 \tag{17}$$

When  $C_1 = 1/4$  the solution ends at the fixed point where  $g' = \varphi' = 0$ . This corresponds to an  $AdS_3$  region where the Riemann surface has constant size. We can see from the equations (14) that this happens when  $e^{2g} = 2^{-\frac{4}{3}}$  and  $e^{3\varphi} = 2$ . We also have then  $e^{2f} = \frac{1}{2^{\frac{4}{3}}r^2}$ . The solution interpolates between  $AdS_5$  and  $AdS_3 \times \Sigma$ . We can calculate the central charge from the supergravity solution by calculating the effective three dimensional Newton's constant and then use the formula  $c = 3R_{ads}/(2G_N^{(3)})$ . In order to do this it is usefull first to calculate the five dimensional Newton's constant and then reduce from five to three dimensions. We are quotienting  $H_2$  so that we have a finite volume Riemann surface. Since the surface has constant curvature we can relate its volume to the radius of curvature of  $H_2$  and the genus. After we do this we get a central charge which agrees with (13). See the appendix for more details.

Once we know the five dimensional metric and gauge fields it is easy to lift up the solutions to ten dimensions using the formulas in [33]. The ten dimensional metric in the  $AdS_3$  region, where  $\varphi$  and g are constant, reads

$$ds^{2} = 2^{-\frac{3}{2}}\sqrt{\Delta}ds_{3}^{2} + \frac{2^{-\frac{3}{2}}}{\sqrt{\Delta}}\left[\Delta\frac{dx^{2} + dy^{2}}{y^{2}} + 2\Delta d\theta^{2} + 4\cos^{2}\theta(d\psi^{2} + \sin^{2}\psi(d\phi_{1} + \frac{1}{2y}dx)^{2} + \cos^{2}\psi(d\phi_{2} + \frac{1}{2y}dx)^{2})) + 2\sin^{2}\theta d\phi_{3}^{2}\right]$$
(18)

where we have defined;

$$ds_3^2 = \frac{(dr^2 - dt^2 + dz^2)}{r^2} \tag{19}$$

$$\Delta = 1 + \sin^2 \theta \tag{20}$$

the angles  $\theta, \psi, \phi_i, i: 1, 2, 3$  were introduced in order to parametrize the five sphere. The expression for the self dual five form  $F_5$  can also be obtained from the formulas in [33]. The metric (18) has  $SO(2) \times SO(2) \times SU(2)$  isometries as expected. The SU(2) isometry is not so obvious in the coordinates in (18) but can be made more manifest by writing the original  $S^3$  parametrized by  $\psi, \phi_1, \phi_2$  as the Hopf fibration on  $S^2$ . Then the gauge field is only appearing in the fiber and the SU(2) isometries are those of  $S^2$ .

It is interesting to note that there are other solutions like the one above, where we allow a second scalar field  $\varphi_2$ . This second scalar field is associated in the original four dimensional field theory to the operator  $\mathcal{O}'_2 \sim |\mathcal{W}_1|^2 - |\mathcal{W}_2|^2$ . In the IR region they describe marginal deformations of the theory with  $\varphi_2 = 0$ . This marginal deformation breaks the SU(2) isometry mentioned above to SO(2). In the  $AdS_3$  region the fields take values that can be parametrized in term of  $\varphi_2$  as (see the appendix for details)

$$e^{3\varphi_1} = 2\cosh\varphi_2$$
  $e^{2g} = \frac{e^{2\varphi_1}}{4}$   $f = -\log[e^{2\varphi_1}r]$  (21)

It is easy to see that the supergravity central charge is the same as above (13).

There are more twistings of the D3 brane theory that we could consider. In the appendix we discuss more general cases. In some of those cases the spins of fields after twisting are not integer of half integer, so it is not clear to us that the theories really make sense.

Some twisted solutions of five dimensional gauged supergravities were considered in [6, 7]. Those solutions had the restriction that the scalar fields had to be constant in the flow from five to three dimensions. This restriction seems hard to obey if we insist in not having fields with fractional spins, at least for the case of a D3 brane.

## 4 Twists of the M5 brane theory

In this section we consider M5 branes wrapped on two dimensional Riemann surfaces  $\Sigma_g$  times  $R^4$ . We take the limit  $l_p \to 0$  keeping the size of the Riemann surface fixed so that we obtain the (0,2) six-dimensional superconformal theory on  $R^4 \times \Sigma_g$ . We will consider two possible ways of embedding the spin conection in the SO(5) R-symmetry group of the (0,2) SCFT. In the first we preserve two supersymmetries and in the second we preserve one supersymmetry (in d=4 notation). It is convenient to use a  $U(1)\times U(1)$  truncation of the SO(5) gauged supergravity theory. If we denote by 1,...,5 the directions orthogonal to the 5-brane, then these two U(1)'s rotate the directions 12 and 34 respectively. This truncation was studied in [33] and the supersymmetry variation was studied in [45]. In both cases we will make the following ansatz for the metric

$$ds^{2} = e^{2f(r)}(dr^{2} + du^{2} + dv^{2} + dz^{2} - dt^{2}) + \frac{e^{2g(r)}}{y^{2}}(dx^{2} + dy^{2})$$
(22)

The boundary condition is as in the previous case that  $g \sim f \sim -log(r)$ , for  $r \to 0$ .

Throughout this section we work in units where the radius of  $AdS_7$  is one. We can restore the dependence on this parameter by multiplying the seven or ten dimensional metric by the radius of  $AdS_7$  which is given by  $R_{AdS_7}^2 = 4(\pi N)^{2/3} l_p^3$ . <sup>5</sup>

#### 4.1 $\mathcal{N} = 2$ case

In this section we consider configurations preserving  $\mathcal{N}=2$  supersymmetry in four dimensions. The field content and the Lagrangian of the d=7 gauged supergravity, together with the relevant supersymmetry transformations are described in detail in [45] and in the appendix.

In order to get an idea of what the field theory looks like let us first consider a single M5 brane. In this case, in analogy with the D3 brane case we expect that there is an operator  $\mathcal{O}_4 = |\mathcal{Z}|^2 - 2/3(\mathcal{W}_3^2 + \mathcal{W}_4^2 + \mathcal{W}_5^2)$  where  $\mathcal{Z} = \mathcal{W}_1 + i\mathcal{W}_2$  is the scalar that we are twisting and  $\mathcal{W}_i$  are the five scalar fields describing fluctuation of the M5 brane in the five orthogonal dimensions. The presence of this operator can be seen by expanding directly the Nambu action for the brane. We can calculate its coefficient as we did for the D3 brane case around (2), (5). In the case of multiple coincident branes we also have an operator of dimension 4, with the same SO(5) transformation properties which is turned on. So we expect that a supergravity field, dual to this operator, will have a non-trivial boundary condition at infinity.

In order to get an idea of what the field theory looks like, let us consider first the case of a single M5 brane. If it is wrapped on  $S^2$  then we only get four massless

<sup>&</sup>lt;sup>5</sup>In conventions were the eleven dimensional Newton's constant is  $G_N^{11} = 16\pi^7 l_p^9$  [23].

modes, the three scalar fields in the directions that are untwisted and the component of the B field on  $S^2$ . These form a single four-dimensional hypermultiplet. This theory therefore has only a "higgs" branch. In the case that the M5 wrapps a genus g > 1 Riemann surface then we get the same hypermultiplet that we were getting above plus g vector multiplets. These vector multiplets correspond to modes of the B field with one index along a non-contractible cycle in the Riemann surface and one index in four dimensions. So we get a  $U(1)^g$  theory with one neutral hypermultiplet. The scalars in the vector multiplet correspond to modes of the twisted scalar field  $\mathcal{Z}$  and they represent deformations of the Riemann surface in the normal directions that are twisted.

It is less clear what the resulting four dimensional gauge theory is when we have N coincident branes, since we do not have an explicit Lagrangian we could use in six dimensions to derive the four dimensional gauge theory. It seems possible to give a DLCQ definition of the theory in the spirit of [46, 47, 48]. Of course, it would be very nice to give a direct definition of the theory. As a step in that direction we compactify one of the worldvolume directions on a circle so that we get a D4 brane wrapped on  $\Sigma$ . Then the low energy theory is a three dimensional U(N) gauge theory with 8 supercharges. In the case of  $S^2$  get get a pure gauge theory with only a Coulomb branch while in the q > 1 case we also get q adjoint hypers. Notice that the four dimensional theory we want to find should be such that when we reduce it on a circle the Higgs branch of the 4d theory should become the Coulomb branch of the 3d theory and the Higgs branch of the 3d theory results from the Coulomb branch of the 4d theory as in [49]. After dimensional reduction to three dimensions Higgs and Coulomb branches would be exchanged [50]. In the case of  $S^2$  we could say a bit more about this four dimensional low energy theory. When we go down to three dimensions we get a U(N) field theory. The Coulomb branch of this theory is given by the moduli space of N monopoles in SU(2) [49, 51]. Three dimensional mirror symmetry exchanges this Coulomb branch with the Higgs branch of the mirror. So the four dimensional theory is a theory whose Higgs branch is the same as the moduli space of N monopoles of SU(2). This space is smooth and has no singularities. So the four dimensional theory is just this sigma model with only hypermultiplets <sup>6</sup>. It is harder to say what the four dimensional field theory is in the case of q > 1. Again we can say fairly easily what the Higgs branch of the four dimensional theory is. It is the same as the Coulomb branch of the 3d theory. It is less clear what the Coulomb branch of the four dimensional theory is. What we know is that upon reducing to three dimensions and doubling the variables, as in [49] we should find the Higgs branch of the D4 theory. The dimensionality of this Higgs branch is what we analyzed for the D3 brane in section 3.1, it has dimension  $4N^2(g-1)$ . So we expect that the Coulomb branch of the four

<sup>&</sup>lt;sup>6</sup>We thank M. Douglas for suggesting this possibility.

dimensional theory has dimension  $2N^2(g-1)$ . This is suggestive of a very large gauge group of the form  $U(N)^{(g-1)N}$ . Below we will find, from a supergravity analysis, that the theory has a conformal fixed point where the effective number of degrees of freedom goes as  $N^3(g-1)$ , which is in rough agreement with the type of gauge group we expect.

Let us now turn to the supergravity solutions. Setting to zero the supersymmetry variations we get

$$g' = -e^{f} \left[ e^{2\lambda} - \kappa \frac{e^{-2g}}{4} e^{3\lambda} \right] + \frac{1}{2} \lambda'$$

$$\lambda' = -\frac{2}{5} e^{f} \left[ 2(e^{-3\lambda} - e^{2\lambda}) + \kappa \frac{1}{4} e^{-2g+3\lambda} \right]$$

$$f' = -e^{f+2\lambda} + \frac{1}{2} \lambda'$$
(23)

where  $\kappa = 1$  for  $H^2$  and  $\kappa = -1$  for  $S^2$ . The general solution of the equations is

$$e^{5\lambda} = \frac{e^{2\rho} + \kappa \frac{1}{2} + C_1 e^{-2\rho}}{e^{2\rho} + \kappa \frac{1}{4}}$$

$$e^{2g} = e^{\lambda} (e^{2\rho} + \kappa \frac{1}{4})$$

$$e^{2f} = C_2 e^{2\rho} e^{\lambda}$$

$$e^{2f} \left(\frac{dr}{d\rho}\right)^2 = g_{\rho\rho} = e^{-4\lambda}$$
(24)

We have chosen a new variable  $\rho$  as implicit in the last equation.  $\rho \to +\infty$  corresponds to the boundary of  $AdS_7$ .  $C_2$  is a trivial integration constant that can be absorbed by rescaling the four dimensional coordinates. It is related to the size of the surface where we are wrapping the six dimensional theory. In the case  $C_1 = 0$   $\kappa = 1$  we find that the solution interpolates between  $AdS_7$  and  $AdS_5 \times \Sigma_{g>1}$ . This is telling us that the IR dynamics of the (0,2) six dimensional theory on  $\Sigma_{g>1} \times R^4$  is given by a four dimensional superconformal field theory. The solution in the IR has the fixed point values

$$e^{5\lambda} = 2$$
,  $e^{2g-\lambda} = \frac{1}{4}$ ,  $e^{f+2\lambda} = \frac{1}{r}$ . (25)

The sign of the expectation value of the operator  $\mathcal{O}_4$  is again given by the sign of  $C_1$ . A positive value of  $C_1$  therefore corresponds to a positive expectation value for the twisted scalars and therefore to the Coulomb branch. A negative value of  $C_1$  corresponds to an expectation value for the scalars that are untwisted and therefore to the Higgs branch. However for the case of  $S^2$  we do not expect a Coulomb branch since the twisted scalars cannot have an expectation value. Fortunately either Gubser's criterion or our criterion in section 5 rules out this case but allows all the other cases. So in the case of

<sup>&</sup>lt;sup>7</sup>For a related discussion see [1].

 $H^2$  we can have both signs on  $C_1$  and in the case of  $S^2$  we find that  $C_1 \leq \frac{1}{16}$ . The fact that we have 1/16 instead of zero is due to the fact that it is difficult to disentangle the expectation value from the insertion of the operator in supergravity so that our field theory expectation are easy to check only for large values of  $|C_1|$ , presumably a more correct analysis valid also for small values of  $C_1$  would still agree with field theory expectations.

Using the formulas in [33] we can uplift the solution to eleven dimensions. Here we give only the form of the solution in its IR fixed point, the  $AdS_5$  region.

$$ds_{11}^{2} = \frac{1}{2} \Delta^{1/3} ds_{AdS_{5}}^{2} + \frac{\Delta^{-\frac{2}{3}}}{4} \left[ \Delta \frac{(dx^{2} + dy^{2})}{y^{2}} + \Delta d\theta^{2} + \cos^{2}\theta (d\psi + \sin^{2}\psi d\phi_{1}^{2}) + 2\sin^{2}\theta (d\phi_{2} + \frac{dx}{y})^{2} \right]$$

$$\Delta = (1 + \cos^{2}\theta)$$
(26)

Here, the angles  $\theta$ ,  $\psi$ ,  $\phi_1$ ,  $\phi_2$  parametrize the four sphere (before the twisting).  $ds_{AdS_5}^2$  denotes a unit radius Anti-de-Sitter metric. Note that (26) has  $SU(2) \times U(1)$  isometries as required by  $\mathcal{N} = 2$  superconformal invariance. An explicit expression for the four form gauge field can be written following section 4 of [33].

This is giving non-singular  $AdS_5$  warped compactifications of M theory. These compactifications are completely smooth. Singular compactifications to  $AdS_5$  were given in [1, 2], see however [3, 22].

It is also possible to find the supergravity duals of the D4 brane theories wrapped on  $\Sigma$  that we discussed above by compactifying the M5 brane solution along a longitudinal direction and then using the standard reduction from eleven dimensional supergravity to type IIA supergravity. We write this solution explicitly in the appendix. In the case of  $S^2$  this solution is related to pure 3d SYM with 8 supercharges. This is the same system that was explored in [52]. In [52] the starting point was a D6 brane wrapping K3. At low energies this reduces to pure SYM in 3d. A puzzling aspect of the solutions in [52] was that the K3 remained in the IR geometry, at least at first sight. If we take a limit of the solution in [52] where the volume of K3 goes to zero, then we can T-dualize and obtain a large K3 and the D6 brane becomes a D4 brane wrapping  $S^2$  which is what we have here. In the IR region of our solution with  $C_1 = 1/16$  we find that the volume of  $S^2$  shrinks to zero much faster than  $g_{00}$  so we expect that KK excitations in this  $S^2$  decouple from the low energy theory. More precisely, dual of large N pure SYM in 3d would be the region close to the singularity. It is possible that there is a better gravity description for the IR region than the one we explored here, or it could be that the theory only admits a string theory dual but not a weakly coupled gravity dual. The excision of the singularity in [52] amounts to our criterion that the singularity is admissible by the criterion in section 5. In fact it was argued in [52] that it was

necessary to move the branes into the Coulomb branch of the 3d theory to remove the singularities.

#### **4.2** $\mathcal{N} = 1$ case

In this section we consider the case where we embed the spin connection in both of the U(1) factors described above. When we wrap an M5 brane on a Riemann surface in a CY space we typically get normal bundles which are topologically equivalent to this one. The resulting four dimensional field theory has 4 supercharges. It seems hard to say explicitly what this four dimensional theory is. Again if we compactify one circle and we go to three dimensions we can say more. In that case we have a D4 brane wrapped on the Riemann surface. In the  $S^2$  case we find just pure U(N) gauge theory with 4 supercharges.

Let us turn to the supergravity analysis. The equations that result from setting to zero the supersymmetry variations are (see the appendix),

$$f' = -e^{4\phi + f} + \phi'$$

$$g' = -e^{f} (e^{4\phi} - \kappa \frac{1}{4} e^{\phi - 2g}) + \phi'$$

$$\phi' = -\frac{1}{5} e^{f} (4(e^{-\phi} - e^{4\phi}) + \kappa \frac{1}{4} e^{\phi - 2g})$$
(27)

where  $\kappa = 1$  for  $H^2$  and  $\kappa = -1$  for  $S^2$ . In the case of  $H^2$ , this set of equations have solutions for constant values of  $\phi$ , g given by

$$e^{5\phi} = \frac{4}{3}$$
,  $e^{-2g-3\phi} = 4$ ,  $e^{f(r)} = e^{-4\phi} \frac{1}{r}$ . (28)

The corresponding eleven dimensional metric is smooth and can be obtained from [33]. Here we give only the  $AdS_5$  region of the solution which can be found using the IR values (28)

$$ds_{11}^{2} = \Delta^{\frac{1}{3}} ds_{7}^{2} + \frac{\Delta^{-\frac{2}{3}}}{4} \left[ e^{-4\phi} d\mu_{0}^{2} + e^{\phi} (d\mu_{1}^{2} + d\mu_{2}^{2} + \mu_{1}^{2} (d\phi_{1} + \frac{1}{2y} dx)^{2} + \mu_{2}^{2} (d\phi_{2} + \frac{1}{2y} dx)^{2} \right]$$

$$\Delta = e^{4\phi} \mu_{0}^{2} + e^{-\phi} (\mu_{1}^{2} + \mu_{2}^{2})$$
(29)

where  $\mu_i$  parameterize an  $S^2$ ,  $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$  and the seven dimensional metric is as in (22) with the values given in (28).

It would be nice to determine whether these  $AdS_5$  compactifications of M-theory are stable under quantum corrections. In other words, it would be interesting to determine whether these field theories are exactly conformal of if they are non-conformal when we take into account 1/N corrections, as the ones in [53].

## 5 A criterion for allowed singularities

In this section we discuss a proposal for a criterion that would tell us if a supergravity singularity in the IR region of a geometry describing a field theory is allowed or not. The strong form of the final criterion is

The  $g_{00}$  component of the metric should not increase as we approach the singularity we will also discuss a weak form of the criterion

The  $g_{00}$  component of the metric should be bounded above.

In what follows we explain this criterion and give a heuristic motivation for it. When we are trying to find supergravity solutions that are dual to field theories we typically encounter singular solutions. These singularities do not necessarily mean that the solutions are wrong, they might be telling us that the supergravity description is failing and that we should go to a dual description. This is the case for D-p-branes for  $p \neq 3$ [24]. It is clear however that not all singularities are allowed. For example a solution like negative mass Schwarzschild should not be allowed since it would imply that the energy is not bounded below. We would like to propose a necessary criterion for weeding out unphysical singularities. Our criterion applies for the IR regions of supergravity backgrounds which are dual to some field theory. If we want to interpret some region as being dual to the IR of some field theory we expect that  $g_{00}$  should decrease so that fixed proper energy excitations correspond to lower and lower energy excitations from the point of view of coordinate time, which is the same as field theory time. So our criterion will be that  $g_{00}$  should not increase as we approach the singularity. In particular it should not go to infinity. In many cases we can approach the singularity in various directions in the internal manifold. We require that  $g_{00}$  does not increase as we approach the singularity along any direction in the internal manifold. Note that it makes sense to talk about  $g_{00}$  since we are talking about a field theory with a time translation isometry, which is generated by a Killing vector of the dual geometry, so we are choosing coordinates so that this vector is  $\frac{\partial}{\partial t}$ .

This criterion certainly forbids negative mass Schwarzschild singularities where  $g_{00} \rightarrow \infty$ . There are some singularities where  $g_{00}$  stays constant, like orbifold singularities. We should certainly allow those. Or course the full string theory will then tell us whether it is really allowed or not. There are some singularities where  $g_{00}$  increases as one approaches the singularity but stays bounded. A D8 brane has a singularity of this form and it looks like it should be allowed in the full string theory but it seems that it should not be allowed in a region that we want to interpret as the IR of some field theory. But in order to be sure we are not over-restrictive we could allow these and we get the weak form criterion stated in the beginning of this section. If we rule these out we get the first. A diverging  $g_{00}$  implies that the singularity is repulsive, massive particles are repelled from the singularity. In these singularities a finite proper energy

excitation will have very high energy from the point of view of the field theory. It seems to violate the UV/IR correspondence. It is also hard to see how these singularities can form from gravitational collapse since they are repulsive. For a concrete example where an example of a repulsive singularity is discussed see [52]. Note that the c theorem in [54] does not imply that  $g_{00}$  always decreases. In [54] it was proven that the metric of gauged supergravity decreases, but the full ten or eleven dimensional  $g_{00}$  factor includes a warp factor and this warp factor could increase even though the metric appearing in the gauged supergravity action is decreasing. The physical redshift factor is given by the ten or eleven dimensional metric, as long as the supergravity solution is valid. In string theory one should use the Einstein metric for this analysis.

In [27] it was proposed that admissible singularities are those that can be generalized to finite temperature. The criterion proposed here is easy to check and it can be applied directly to the ten or eleven dimensional metric and does not need to use the gauged supergravity form of the potential. In the cases analyzed here it gives the same results as Gubser's criterion [27] we suspect that the boundedness of the gauged supergravity potential will ensure that  $g_{00}$  is bounded but we did not prove it.

Note that this criterion is valid for regions that we want to interpret as the IR of a gauge theory. It is certainly violated when we approach the boundary of AdS or the boundaries of the geometries describing some D-p-branes [24]. In fact it is necessary to put boundary conditions for fields in the boundary of AdS. These boundary conditions are interpreted as defining the details (operator insertions) of the field theory dual to the given background.

## 6 A no go theorem

In this section, which is to great extent disconnected with the rest of the paper, we present an argument saying that there are no non-singular wrapped compactifications in a large class of supergravity theories. Our main assumption will be that the potential for scalar fields is non-positive. The massive type IIA case is treated separately. We do not need to use the equations of motion of the matter fields. Then we will show that there is no non-singular warped compactification to  $R^d$  or de-Sitter space  $dS^d$ , but the reader interested in real world applications might want to take d=4. The no go theorem remains true even in the case that we allow singularities such as those allowed by the strong form of the criterion in section 5. These are singularities that we might be able to interpret as arising from the IR dynamics of some field theory. The argument is quite general and only relies on the equation of motion for the warp factor and does not rely on supersymmetry. There are ways to evade this argument which involve including higher derivative corrections to the supergravity equations, or

starting from a theory that already has a positive cosmological constant. There are other no go theorems for supersymmetric RS models using five dimensional gauged supergravity [12, 11, 13]. These are complementary to our arguments, since the five dimensional potential could be positive. If the five dimensional gauged supergravity arises from a large volume compactification then we could apply our results but the arguments in [12, 11, 13] also cover 5d theories which cannot be interpreted as large volume compactifications.

We consider a D dimensional gravity theory, with D > 2, compactified down to d dimensions. We denote by M, N, L, ... the D dimensional indices. We denote by  $\nu, \nu, \rho, ...$  the d dimensional indices and by m, n, l, the D - d dimensional indices. We will assume that the D dimensional gravity theory satisfies the following conditions.

- The gravity action does not contain higher curvature corrections.
- The potential is non-positive,  $V \leq 0$ . This condition in not obeyed in massive IIA supergravity which has a positive cosmological constant so we treat that case separately in 6.3. V could be just a negative cosmological constant or it can depend on the scalars but it cannot be positive (at least in the range of values of scalar fields that is explored in the solution under consideration).
- The theory contains massless fields with positive kinetic terms. These massless fields have field strengths which are n forms,  $F_{i_1,...,i_n}$ . For n=1 we have scalar fields, n=2 Maxwell fields (these could be non-abelian, as long as the metric on the group is positive definite so that the kinetic terms are positive), etc. We consider n < D, if n=D it would give a contribution similar to a potential and we go back to the previous assumption.
- The d dimensional effective Newton's constant is finite.

We start by writing out Einstein's equations in D dimensions

$$R_{MN} = T_{MN} - \frac{1}{D - 2} g_{MN} T_L^L \tag{30}$$

Notice that in (32) we neglected higher derivative corrections. We write the metric as

$$ds_D^2 = \Omega^2(y) \left( dx_d^2 + \hat{g}_{mn} dy^n dy^m \right)$$
(31)

where  $dx_d^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu}$  where  $\eta$  is the metric of the d dimensional space which is either Minkowski or de-Sitter space. Now we calculate the  $R_{\mu\nu}$  components of the D dimensional metric and we find that Einstein's equations imply

$$R_{\mu\nu} = R_{\mu\nu}(\eta) - \eta_{\mu\nu} \left( \hat{\nabla}^2 \log \Omega + (D - 2)(\hat{\nabla} \log \Omega)^2 \right) = T_{\mu\nu} - \frac{1}{D - 2} \Omega^2 \eta_{\mu\nu} T_L^L$$
 (32)

where the hat denotes covariant derivatives and contraction of indices with respect to the metric  $\hat{g}$ . Taking the trace over  $\eta$  on both sides we find

$$\hat{\nabla}^2 \log \Omega + (D-2)(\hat{\nabla} \log \Omega)^2 = \frac{1}{(D-2)\Omega^{D-2}} \nabla^2 \Omega^{D-2} = R(\eta) + \Omega^2 (-T^{\mu}_{\ \mu} + \frac{d}{D-2} T^L_{\ L})$$
(33)

where in the term involving the stress tensor on the right hand side we contract the indices with the D dimensional metric and  $R(\eta)$  is the curvature of the d dimensional metric  $\eta$ . We will now proceed to prove that the term in the right hand side involving the stress tensor is non-negative.

## **6.1** $\tilde{T} \geq 0$

The stress tensor will be the sum of the contributions to the stress tensor of the various massless fields. We will consider each contribution individually since they are all adding up to the total stress tensor. Let us define

$$\tilde{T} \equiv -T^{\mu}_{\ \mu} + \frac{d}{D-2}T^{L}_{\ L} \tag{34}$$

We want to show that all contributions to  $\tilde{T}$  are non-negative. Let us first consider the potential term. We will not keep track of irrelevant positive numerical constants. The stress tensor is

$$T_{MN} \sim -V g_{MN} , \qquad \tilde{T} \sim -V \frac{2d}{D-2} \ge 0$$
 (35)

if V<0 as assumed. Now let us consider the n form field strengths. Their stress tensors are

$$T_{MN} = F_{ML_1..L_{n-1}} F_N^{L_1..L_{n-1}} - \frac{1}{2n} g_{MN} F^2$$

$$\tilde{T} = -F_{\mu L_1..L_{n-1}} F^{\mu L_1..L_{n-1}} + \frac{d}{D-2} (1 - \frac{1}{n}) F^2$$
(36)

In principle we could have functions of scalar fields multiplying these expressions, as we have in some supergravity theories, and we could also have many types of n form fields. We will not indicate these explicitly but it is obvious how to extend the following arguments to those cases. The space time indices of non-vanishing components of F could be completely along the internal dimensions or, if  $n \geq d$ , they could have d out of n indices along the d dimensions and the rest along the internal dimensions. Other possibilities do not preserve the isometries of  $R^d$  or  $dS^d$ . In constructing  $\tilde{T}$  these two types of components will make separate contributions. We will therefore consider them independently and show that each of them is positive. So let us first consider the part of F with all indices internal. Then we have that  $F^2 \geq 0$  and we see from (36) that we have a positive contribution. For all n > 1 forms this contribution is strictly positive

if we have a non-vanishing field strength, but for n=1 the contribution is zero even if we have a non-vanishing field strength. Now we consider the part of the field strength with components along the d-dimensional space. The difference between the term that contains a trace over the  $\mu$  index and the others is that we are choosing a particular order of contractions of the indices comparing the two we find that

$$F_{\mu L_1 \dots L_{n-1}} F^{\mu L_1 \dots L_{n-1}} = \frac{d}{n} F^2.$$

Then we find that

$$\tilde{T} = -F^2 \frac{d(D-1-n)}{n(D-2)} \ge 0 \tag{37}$$

Where we used that  $F^2 < 0$  since we are considering temporal components of F. We have also used that we are considering  $n \le D - 1$ .

#### 6.2 Condition on the warp factor

Multiplying (33) by a power of  $\Omega$  and using that  $\tilde{T} \geq 0$  we conclude that

$$\hat{\Omega}^{(D-2)} \nabla^2 \Omega^{(D-2)} \ge 0 \tag{38}$$

with equality holding only if the right hand side of (33) is zero so that the d dimensional space is Minkowski space. Remember that the d dimensional Newton constant is given by

$$\frac{1}{G_N^d} \sim \int d^d y \sqrt{\hat{g}} \Omega^{(D-2)} \tag{39}$$

We are assuming that this Newton constant is finite.

Let us first assume that  $\Omega$  is bounded below and above in the internal manifold. In that case the internal manifold should be compact. Integrating (38) over the compact internal space by parts we conclude that  $\int d^(D-d)y\sqrt{\hat{g}}(\hat{\nabla}\Omega^{(D-2)})^2 \leq 0$  which is possible only if  $\Omega$  is constant. In that case we conclude that the right hand side of (33) is zero, so that we cannot have a deSitter space and the only n forms that we can be turned on are the n=1,D-1 forms.

As discussed in section 5 we expect that singularities where  $\Omega$  diverges should not be allowed. So we conclude that  $\Omega$  is bounded above. Now suppose that we have regions where  $\Omega \to 0$  or we allow singularities obeying the strong form of the criterion in section 5, which says that  $g_{00}$  should not increase as we approach the singularity. In this case we can define a region  $\mathcal{R}$  which leaves out the singularities and such that  $\Omega > \epsilon$  in  $\mathcal{R}$  for a suitably small  $\epsilon$ . By our assumptions about the singularities it is clear that we can choose  $\mathcal{R}$  so that  $\nabla\Omega$  is either zero or pointing inwards at the boundary of  $\mathcal{R}$ .

Now we can integrate (38) by parts in region  $\mathcal{R}$ , and we get

$$\int_{\mathcal{R}_{\epsilon}} \left( \hat{\nabla} \Omega^{(D-2)} \right)^2 \le -\int_{\partial \mathcal{R}_{\epsilon}} \left( \vec{n} \nabla \Omega^{(D-2)} \right) \Omega^{(D-2)} \le 0 \tag{40}$$

where we used the assumption that  $\Omega$  was non-increasing at the boundary. Again we conclude that the warp factor has to be constant and that a R-S or deSitter compactification of this type is not allowed. <sup>8</sup>.

In summary we have proven that given our assumptions there are no compactifications to deSitter space or Randall-Sundrum compactifications where the only possible singularities are such that the warp factor  $\Omega$  going to zero at the singularity. As we explained in section 5, these are the singularities which might have a field theory interpretation.

The fact that de-Sitter is not allowed *does not* imply that there are no expanding universe solutions in large volume compactifications, it only says that there are no homogeneous SO(1, d+1) invariant de-Sitter solutions. So we could have solutions where some scalar fields are time dependent.<sup>9</sup>

The most natural question is whether there are ways to evade this argument. For that we note that once we include higher derivative corrections to the gravity action, like the ones present in string theory or M-theory, then the positivity argument does not hold and there can be warped compactifications [57, 58, 59, 60, 61, 62]. For example, in heterotic compactifications or Horava Witten compactifications [63] we can have warp factors but we need crucially the higher derivative terms which modify the Bianchi identity for the three or four form field strengths respectively. The same can be said about type I examples where we have orientifolds. These higher derivative terms are crucial to get important physical aspects of string compactifications. All we are saying is that these stringy corrections to the gravity equations are also crucial if one wants a deSitter of Randall Sundrum compactification [10]. Of course, it is very interesting to study these solutions more precisely. It is possible to evade the arguments in this section by by allowing potentials for scalar fields that are positive, as explicitly demonstrated in [64, 65].

<sup>&</sup>lt;sup>8</sup>Note that the solutions in [55] do not obey the condition that the higher dimensional  $g_{00}$  goes to zero at the singularity.

<sup>&</sup>lt;sup>9</sup>The same comment applies to the statement in [56] that their compactifications have no de-Sitter or Anti-de-Sitter solutions. On general grounds we expect to find also expanding or contracting universes with time dependent scalar fields containing singularities similar to the ones considered in [56].

# 6.3 No Minkowski or de Sitter compactifications of massive IIA sugra

In this section we show that there are no compactification of massive IIA on smooth manifolds without boundaries down to  $R^d$  or  $dS^d$ . The equations of motion for the metric and the dilaton in massive IIA supergravity are given by [66]

$$R_{MN} = \frac{m^2}{16} e^{-5\phi} g_{MN} + 2\partial_M \phi \partial_N \phi + e^{2\phi} (G_M^{PQ} G_{NPQ} - \frac{1}{12} g_{MN} G^2) + + 2m^2 e^{-3\phi} (B_M^{P} B_{NP} - \frac{1}{16} g_{MN} B^2) + \frac{1}{3} e^{-\phi} (F_M^{PQR} F_{NPQR} - \frac{3}{32} g_{MN} F^2)$$
(41)

$$0 = \Box \phi + \frac{5m^2}{8}e^{-5\phi} + \frac{1}{48}e^{-\phi}F^2 - \frac{1}{6}e^{2\phi}G^2 + \frac{3m^2}{4}e^{-3\phi}B^2$$
 (42)

where we have defined the square of a tensor of n indices as  $H^2 = H^{M_1...M_n}H_{M_1...M_n}$ . We will not need the precise definition of the tensors B, G and F in (41)(42) but we will use that they are real and antisymmetric. We assume  $d \geq 2$ . We make an ansatz for the metric as in (31). Following the steps that lead to (33) we find

$$\frac{1}{(D-2)\Omega^{D}} \nabla^{2} \Omega^{D-2} = \Omega^{-2} R(\eta) + d \left[ -\frac{m^{2}}{16} e^{-5\phi} + \frac{1}{12} e^{2\phi} G_{e}^{2} + \frac{m^{2}}{8} e^{-3\phi} B_{e}^{2} + \frac{1}{32} e^{-\phi} F_{e}^{2} \right] - d \left[ \theta(3-d) \frac{1}{4} e^{2\phi} G_{l}^{2} + \theta(2-d) \frac{7m^{2}}{8} B_{l}^{2} + \theta(4-d) \frac{5}{96} e^{-\phi} F_{l}^{2} \right]$$
(43)

where we have denoted by  $H_e^2$  the square of a tensor with components purely in the internal dimensions and by  $H_l^2$  the square of the tensor with components along the d dimensional spacetime directions. These can only appear if the rank of the tensor is bigger or equal to d that is the reason we have factors of  $\theta(n-d)$  where  $\theta(x)=1$  for  $x\geq 0$  and zero otherwise. Now notice that

$$\Box \phi = \frac{1}{\Omega^D} \hat{\nabla}_m \left( \Omega^{D-2} \hat{g}^{mn} \partial_n \phi \right) \tag{44}$$

We then see that if we multiply (43) by  $10\Omega^D$  and add (44) times  $d\Omega^D$  we get

$$10\Omega^{D} \times (43) + d\Omega^{D} \times (42) = \frac{10}{D-2} \hat{\nabla}^{2} \Omega^{D-2} + d\hat{\nabla}_{m} \left( \Omega^{D-2} \hat{g}^{mn} \partial_{n} \phi \right) =$$

$$= \Omega^{D-2} R(\eta) + d\Omega^{D} \left[ \frac{2}{3} e^{2\phi} G_{e}^{2} + 2m^{2} e^{-3\phi} B_{e}^{2} + \frac{1}{3} e^{-\phi} F_{e}^{2} - \right.$$

$$\left. -\theta (3-d) \frac{8}{3} e^{2\phi} G_{l}^{2} - \theta (2-d) 8m^{2} B_{l}^{2} - \theta (4-d) \frac{1}{2} e^{-\phi} F_{l}^{2} \right]$$

$$(45)$$

We see that the right hand side of (45) is positive since  $H_e^2 > 0$  and  $H_l^2 < 0$  for all n forms.

If we have a compact internal manifold then we can integrate (45)over the manifold. We get zero from the left hand side since we have a total derivative. On the right hand side we get a non-zero result unless  $R(\eta) = 0$  and B = F = G = 0. Now that we know this we can integrate just the equation for the dilaton (42) over the internal manifold and we get a contradiction. So we find that there are no non-singular compactifications (over a compact internal manifold) to either de Sitter or Minkowski space. If we assume that we have regions where the warp factor could go to zero but the dilaton stays constant, as we expect in an AdS region. Then we also get a contradiction by following steps similar to those in the previous subsection and including boundary terms for the conformal factor. This excludes compactifications of the RS type. In [66] there are several examples of compactifications to Anti-de-Sitter manifolds.

Obviously there are compactifications of massive IIA to Minkowski space on spacetimes with boundaries where the conformal factor is not decreasing as we approach the boundaries as explicitly demonstrated in [67]. It would be nice to see if we can get compactifications to de Sitter space in cases where the compact manifold has boundaries.

## Acknowledgements

We would like to thank M. Bershadsky, M. Douglas, J. Gomis, K. Hori, C. Johnson, A. Kapustin, A. Strominger and C. Vafa for discussions.

The research of C.N. was supported by CONICET from Argentina and the Antorchas foundation. The research of J.M. was supported in part by DOE grant DE-FGO2-91ER40654, NSF grant PHY-9513835, the Sloan Foundation and the David and Lucile Packard Foundation.

## 7 Appendix

Here we discuss some details in obtaining the supergravity solutions.

#### 7.1 D=5

We start with IIB supergravity in  $AdS_5 \times S^5$ . It is believed that we can obtain a consistent truncation to  $\mathcal{N}=8$  supergravity.  $\mathcal{N}=8$  supergravity involves an SO(6) gauge field. We can further truncate this theory to supergravity theories involving a smaller gauge group. The advantage of considering these truncations is a simplification in the equations of motion. The truncation that we used is a truncation to a supergravity theory in five dimensions with three U(1) gauge fields and two scalar fields

described in [68],[33]. This is an  $\mathcal{N}=2$  supergravity theory in five dimensions with two vector multiplets. The third gauge field is the graviphoton. The three U(1) gauge fields are associated to rotations on the 12, 34 and 56 planes respectively, where 1-6 are directions orthogonal to the D3 brane.

The Lagrangian for this theory is given by [33] [6]) is

$$L = R - \frac{1}{2} (\partial_{\mu} \phi_{1})^{2} - \frac{1}{2} (\partial_{\mu} \phi_{2})^{2} + 4 \sum_{i}^{3} e^{\alpha_{i}} - \frac{1}{4} (\sum_{i}^{3} e^{2\alpha_{i}} F^{i}_{\mu\nu}^{2}) + \frac{1}{4} \epsilon^{\mu\nu\alpha\beta\rho} F^{1}_{\mu\nu} F^{2}_{\alpha\beta} A^{3}_{\rho}$$
(46)

where we have defined

$$\alpha_1 = \frac{\phi_1}{\sqrt{6}} + \frac{\phi_2}{\sqrt{2}}, \quad \alpha_2 = \frac{\phi_1}{\sqrt{6}} - \frac{\phi_2}{\sqrt{2}}, \quad \alpha_3 = \frac{-2\phi_1}{\sqrt{6}},$$
 (47)

We work in units where  $R_{AdS_5} = 1$  for the usual  $AdS_5$  solution, related to  $\mathcal{N} = 4$  SYM. The corresponding supersymmetry transformations are

$$\delta\lambda_{i} = \left(\frac{3}{8}\partial_{i}X_{I}\Gamma^{\mu\nu}F^{I}_{\mu\nu} - \frac{i}{2}g_{ij}\Gamma^{\mu}\partial_{\mu}\phi^{j} + \frac{3i}{2}V_{I}\partial_{i}X^{I}\right)\epsilon$$

$$\delta\psi_{\mu} = \left(D_{\mu} + \frac{i}{8}X_{I}(\Gamma^{\nu\rho}_{\mu} - 4\delta^{\nu}_{\mu}\Gamma^{\rho})F^{I}_{\nu\rho} + \frac{1}{2}\Gamma_{\mu}X^{I}V_{I} - \frac{3i}{2}V_{I}A^{I}_{\mu}\right)\epsilon$$

$$(48)$$

we have defined

$$X^{(1)} = e^{-\frac{\phi_1}{\sqrt{6}} - \frac{\phi_2}{\sqrt{2}}} \qquad X^{(2)} = e^{-\frac{\phi_1}{\sqrt{6}} + \frac{\phi_2}{\sqrt{2}}} \qquad X^{(3)} = e^{2\frac{\phi_1}{\sqrt{6}}} \tag{49}$$

$$X_{(1)} = \frac{1}{3}e^{\frac{\phi_1}{\sqrt{6}} + \frac{\phi_2}{\sqrt{2}}} \qquad X_{(2)} = \frac{1}{3}e^{\frac{\phi_1}{\sqrt{6}} - \frac{\phi_2}{\sqrt{2}}} \qquad X_{(3)} = \frac{1}{3}e^{-2\frac{\phi_1}{\sqrt{6}}}$$
 (50)

$$g_{ij} = \frac{1}{2}\delta_{ij} \qquad V_I = \frac{1}{3} \tag{51}$$

For the metric (7), we have the spin connection

$$\omega_{\hat{x}}^{xy} = -\frac{1}{y} \quad \omega_{\hat{x}}^{xr} = \omega_{\hat{y}}^{yr} = \frac{e^{g-f}g'}{y}, \quad \omega_{\hat{z}}^{zr} = \omega_{\hat{t}}^{tr} = f',$$
 (52)

(here the hatted index is curved and the others are flat) and with the choice of gauge fields

$$A_x^{(1)} = \frac{a}{y} \quad A_x^{(2)} = \frac{b}{y} \quad A_x^{(1)} = \frac{c}{y}$$
 (53)

we have the equations

$$\partial_x \epsilon + \frac{1}{2} \omega_{\hat{x}}^{xy} \Gamma_{xy} \epsilon + \frac{1}{2} \omega_{\hat{x}}^{xr} \Gamma_{xr} \epsilon - \frac{i}{2} X_I F_{xy}^I y e^{-g} \Gamma_y \epsilon + \frac{e^g}{2y} X^I V_I \Gamma_x \epsilon - \frac{i}{2} (A_x^{(1)} + A_x^{(2)} + A_x^{(3)}) \epsilon = 0$$

$$(54)$$

$$\partial_z \epsilon + \frac{1}{2} \omega_{\hat{z}}^{zr} \Gamma_{zr} \epsilon + \frac{i}{4} X_I F_{xy}^I e^{-2g} y^2 \Gamma_{zxy} \epsilon + \frac{1}{2} e^f X^I V_I \gamma_z \epsilon = 0$$
 (55)

$$\frac{3}{4}\partial_{\phi 1}X_{I}F_{xy}^{I}e^{-2g}y^{2}\Gamma_{xy}\epsilon - \frac{i}{4}e^{-f}\phi_{1}'\Gamma_{r}\epsilon + \frac{i}{2}\partial_{\phi 1}(X^{(1)} + X^{(2)} + X^{(3)})\epsilon = 0$$
 (56)

$$\frac{3}{4}\partial_{\phi 2}X_{I}F_{xy}^{I}e^{-2g}y^{2}\Gamma_{xy}\epsilon - \frac{i}{4}e^{-f}\phi_{2}'\Gamma_{r}\epsilon + \frac{i}{2}\partial_{\phi 2}(X^{(1)} + X^{(2)} + X^{(3)})\epsilon = 0$$
 (57)

and another equation describing the radial dependence of the spinor that we do not write here. We impose the following condition on the spinors

$$\Gamma_{xy}\epsilon = -i\beta\epsilon \qquad \Gamma_r\epsilon = \eta\epsilon$$
 (58)

where  $\beta = \pm 1$ ,  $\eta = \pm 1$ . By doing simply parity transformations we set  $\beta = \eta = 1$ . We obtain, from eq. (54)

$$1 = (a+b+c) \tag{59}$$

$$g' = -2e^{f} \left[ \frac{1}{6} (X^{(1)} + X^{(2)} + X^{(3)}) - \frac{1}{2} X_{I} a^{I} e^{-2g} \right]$$
(60)

Equations (55)(56) (57) reduce to

$$f' = -2e^{f} \left[ \frac{1}{6} (X^{(1)} + X^{(2)} + X^{(3)}) + \frac{1}{2} X_{I} a^{I} e^{-2g} \right]$$
 (61)

$$\phi_1' = 4e^f \left[ \frac{1}{2} \partial_{\phi_1} (X^{(1)} + X^{(2)} + X^{(3)}) - \frac{3}{4} \partial_{\phi_1} X_I a^I e^{-2g} \right]$$
 (62)

$$\phi_2' = 4e^f \left[ \frac{1}{2} \partial_{\phi_2} (X^{(1)} + X^{(2)} + X^{(3)}) - \frac{3}{4} \partial_{\phi_2} X_I a^I e^{-2g} \right]$$
(63)

It is easy to see that, the choice

$$\phi_2 = 0, a = b \tag{64}$$

solves (63). Using the explicit expressions for the scalar fields  $X^I$  and defining  $\varphi = \frac{\phi}{\sqrt{6}}$  we obtain

$$g' = -\frac{1}{3}e^{f-g}[e^g(2e^{-\varphi} + e^{2\varphi}) - e^{-g}(2ae^{\varphi} + ce^{-2\varphi})]$$
(65)

$$f' = -\frac{1}{6} \left[ 2e^f (2e^{-\varphi} + e^{2\varphi}) + e^{f-2g} (2ae^{\varphi} + ce^{-2\varphi}) \right]$$
 (66)

$$\phi' = \frac{4}{\sqrt{6}} e^f [(-e^{-\varphi} + e^{2\varphi}) - \frac{1}{2} e^{-2g} (ae^{\varphi} - ce^{-2\varphi})]$$
 (67)

$$2a + c = 1 \tag{68}$$

In the case a=b=0; c=1 we obtain eqs. (8), while for a=b=1/2 we obtain eqs. (14) - (16) .

It can be seen that solutions with constant  $g, \phi$  can be obtained

$$a = b , c = 1 - 2a , e^{\frac{\phi}{\sqrt{6}}} = \left(4 - \frac{1}{a}\right)^{\frac{1}{3}}$$

$$e^{-2g} = \frac{\left(4 - \frac{1}{a}\right)^{\frac{1}{3}}}{a} = \frac{e^{\varphi}}{a}$$

$$e^{f} = e_{0}^{f} \frac{1}{r} , e_{0}^{f} = \frac{2a(4 - \frac{1}{a})^{\frac{1}{3}}}{(6a - 1)}$$

$$(69)$$

A simple analysis of the eqs. (65)-(68) near r = 0, shows that

$$g(r) = -Log(r) + \frac{7(2a+c)}{36}r^2 + \dots$$

$$f(r) = -Log(r) - \frac{(2a+c)}{18}r^2 + \dots$$

$$\phi(r) = -\frac{(a-c)}{3}r^2Log(r) + \dots$$
(70)

The last equation shows that the operator dual to the field  $\phi$  is turned on. The field  $\phi$  is dual to an operator of dimension  $\Delta=2$ . For this dimension the two solutions of the wave equation go as  $\phi \sim r^2$ ,  $\phi \sim r^2 \log(r)$ . The second solution is the non-normalizable mode associated to the insertion of an operator. We see that in both cases analyzed in this paper the operator  $\phi$  is turned on. It is interesting that there is also a special solution where a=b=c=1/3 where the operator  $\phi$  is not turned on. The field  $\phi=0$  in the whole solution. This solution is of the form of the solutions analyzed in [7]. It would be interesting to see if this solution really makes sense since some fields acquire fractional spins with this twisting. In this case the metric has the form [7]

$$ds^{2} = e^{-g} (3e^{2g} - 1)^{\frac{3}{2}} (-dt^{2} + dz^{2}) + \frac{9}{(3 - e^{-2g})^{2}} dg^{2} + \frac{e^{2g}}{y^{2}} (dx^{2} + dy^{2})$$
(71)

We now give some details on the computation of the central charge for the solutions of the form (69) from the supergravity side. We have

$$c = \frac{3R_{AdS3}}{2G_N^3} \tag{72}$$

and

$$\frac{1}{G_N^3} = \frac{Vol_7}{G_N^{10}} = \frac{2N^2}{\pi} Vol_{12}$$
 (73)

where we used that in the units used in our paper where  $R_{AdS_5} = 1$  we have that  $G_N^{10} = \frac{\pi^4}{2N^2}$  and that the volume of a unit radius five-sphere is  $\pi^3$ .

In the case of the solutions (69)  $R_{AdS3} = e^{f_0}$ . We can calculate the volume of a constant curvature Riemann surface as follows

$$\int_{\Sigma_2} \sqrt{g} R = 8\pi (1 - g) \; ; \qquad R = -2e^{-2g} \qquad \to \qquad \int_{\Sigma} \sqrt{g} = 4\pi (g - 1)e^{2g} \tag{74}$$

$$c = 12N^{2}(g-1)e^{2g+f_0} = \frac{24a^{2}N^{2}(g-1)}{6a-1}$$
(75)

for the case  $a = \frac{1}{2}$  we reproduce (13).

As we pointed out in the paragraph before eq. (21) there are other solutions for the case in which we excite two gauge fields  $A_x^{(1)} = \frac{a}{y}$ ,  $A_x^{(2)} = \frac{b}{y}$  and we keep both scalar fields  $\phi_1$ ,  $\phi_2$  nonvanishing.

in that case the eqs. to solve read

$$g' = -\frac{1}{3}e^{f}\left[\left(e^{-\frac{\phi_{1}}{\sqrt{6}} - \frac{\phi_{2}}{\sqrt{2}}} + e^{-\frac{\phi_{1}}{\sqrt{6}} + \frac{\phi_{2}}{\sqrt{2}}} + e^{2\frac{\phi_{1}}{\sqrt{6}}}\right) - e^{-2g}\left(ae^{\frac{\phi_{1}}{\sqrt{6}} + \frac{\phi_{2}}{\sqrt{2}}} + be^{\frac{\phi_{1}}{\sqrt{6}} - \frac{\phi_{2}}{\sqrt{2}}} + ce^{-2\frac{\phi_{1}}{\sqrt{6}}}\right)$$
(76)

$$f' = -\frac{1}{6}e^{f}\left[2\left(e^{-\frac{\phi_{1}}{\sqrt{6}} - \frac{\phi_{2}}{\sqrt{2}}} + e^{-\frac{\phi_{1}}{\sqrt{6}} + \frac{\phi_{2}}{\sqrt{2}}} + e^{2\frac{\phi_{1}}{\sqrt{6}}}\right) + e^{-2g}\left(ae^{\frac{\phi_{1}}{\sqrt{6}} + \frac{\phi_{2}}{\sqrt{2}}} + be^{\frac{\phi_{1}}{\sqrt{6}} - \frac{\phi_{2}}{\sqrt{2}}} + ce^{-2\frac{\phi_{1}}{\sqrt{6}}}\right)$$
(77)

$$\phi_1' = \frac{1}{\sqrt{6}} e^f \left[ 2\left( -e^{-\frac{\phi_1}{\sqrt{6}} - \frac{\phi_2}{\sqrt{2}}} - e^{-\frac{\phi_1}{\sqrt{6}} + \frac{\phi_2}{\sqrt{2}}} + 2e^{2\frac{\phi_1}{\sqrt{6}}} \right) - e^{-2g} \left( ae^{\frac{\phi_1}{\sqrt{6}} + \frac{\phi_2}{\sqrt{2}}} + be^{\frac{\phi_1}{\sqrt{6}} - \frac{\phi_2}{\sqrt{2}}} - 2ce^{-2\frac{\phi_1}{\sqrt{6}}} \right) \right]$$
(78)

$$\phi_2' = \frac{1}{\sqrt{2}} e^f \left[ 2\left( -e^{-\frac{\phi_1}{\sqrt{6}} - \frac{\phi_2}{\sqrt{2}}} + e^{-\frac{\phi_1}{\sqrt{6}} + \frac{\phi_2}{\sqrt{2}}} \right) - e^{-2g} \left( ae^{\frac{\phi_1}{\sqrt{6}} + \frac{\phi_2}{\sqrt{2}}} - be^{\frac{\phi_1}{\sqrt{6}} - \frac{\phi_2}{\sqrt{2}}} \right) \right]$$
(79)

A constant solution to these equations, can be obtained iff  $a=b=\frac{1}{2}$  and reads, after redefining  $\varphi_1=\frac{\phi_1}{\sqrt{6}}$  and  $\varphi_2=\frac{\phi_2}{\sqrt{2}}$ 

$$f[r] = -Log[e^{2\varphi_1}r], \qquad e^{2g} = \frac{e^{2\varphi_1}}{4}, \qquad 2\cosh\varphi_2 = e^{3\varphi_1}$$
 (80)

We end this subsection by noting that according to [33] the  $g_{00}^{(10)}$  component of the ten dimensional metric is related to the five dimensional  $g_{00}^{(5)}$  metric appearing in (46) through

$$g_{00}^{(10)} = W g_{00}^{(5)} , \qquad W^2 = \left( e^{-\frac{\phi_1}{\sqrt{6}} - \frac{\phi_2}{\sqrt{2}}} \sin^2 \psi + e^{-\frac{\phi_1}{\sqrt{6}} + \frac{\phi_2}{\sqrt{2}}} \cos^2 \psi \right) \cos^2 \theta + e^{2\frac{\phi_1}{\sqrt{6}}} \sin^2 \theta . \tag{81}$$

#### 7.2 Effective Potential

Here we consider the effective action for the configurations studied above. Start by considering the Action

$$S = \int d^5x \sqrt{G} [R - \frac{1}{2} \partial \phi^2 - V - \frac{1}{4} \Sigma e^{2\alpha_i} F_{\mu\nu,i}^2]$$
 (82)

with  $\alpha_i$  as in (47) and

$$V = -4\sum_{i}^{3} e^{\alpha_i}.$$
 (83)

Now consider a compactification to three dimensions on the space

$$ds_5^2 = ds_3^2(r, z, t) + \frac{e^{2g}}{y^2}(dx^2 + dy^2) \qquad F_{x,y,i} = c_i/y^2, \qquad \phi_2 = 0$$
 (84)

where we are thinking that we will quotient the  $H^2$  space to obtain a finite volume region. We get

$$S \sim \int d^3x \sqrt{g_3} e^{2g} \left[ R(g_3) - \frac{1}{2} g^{rr} (\partial_r \phi)^2 - 2e^{-2g} - V - \frac{1}{2} \Sigma e^{2\alpha_i} c_i^2 e^{-4g} \right]. \tag{85}$$

Transforming to the Einstein frame

$$g_3 = e^{-4g} \hat{g}_3 \tag{86}$$

we find

$$S = \int d^3x \sqrt{\hat{g}_3} [R(\hat{g}_3) - \frac{1}{2}g^{rr}(\partial_r \phi)^2 - V_{eff}]$$
 (87)

where,

$$V_{eff} = e^{-4g} \left( \pm 2e^{-2g} + V + \frac{e^{-4g}}{2} \left( e^{2\frac{\phi}{\sqrt{6}}} a^2 + e^{2\frac{\phi}{\sqrt{6}}} b^2 + e^{-4\frac{\phi}{\sqrt{6}}} c^2 \right) \right)$$
(88)

where + is for genus q > 1 and - is for  $S^2$ .

#### 7.3 D=7

Again in this case it is convenient to choose a truncation of SO(5) gauged supergravity to a gauged supergravity containing only two U(1) gauge fields. These two U(1) fields correspond to rotations in the 12 and 34 planes, where 1-5 are directions orthogonal to the M5 brane. We use [45] for the supersymmetry transformations and [33] for lifting up the solutions to eleven dimensions. The effective Lagrangian is [45]

$$L = R - 5\partial(\lambda_1 + \lambda_2)^2 - \partial(\lambda_1 - \lambda_2)^2 - e^{-4\lambda_1}F_{\mu\nu,1}^2 - e^{-4\lambda_2}F_{\mu\nu,2}^2$$
$$-\frac{1}{2}m^2(-8e^{2(\lambda_1 + \lambda_2)} - 4e^{-2\lambda_1 - 4\lambda_2} - 4e^{-4\lambda_1 - 2\lambda_2} + e^{-8(\lambda_1 + \lambda_2)}) + L[C_3 = 0] \quad (89)$$

Again we will work in units where  $R_{AdS_7} = 1$  for the usual M5 solution. In these units the radius of  $S^4$  is 1/2 and m = 2 above.

The supersymmetry transformations can be found in [45],

$$\delta\psi_{\mu} = \left[\nabla_{\mu} + k/2(A_{\mu}^{(1)}\Gamma^{12} + A_{\mu}^{(2)}\Gamma^{34}) + \frac{m}{4}e^{-4(\lambda_{1}+\lambda_{2})}\gamma_{\mu} + \frac{1}{2}\gamma_{\mu}\gamma^{\nu}\partial_{\nu}(\lambda_{1}+\lambda_{2}) + \frac{1}{2}\gamma^{\nu}(e^{-2\lambda_{1}}F_{\mu\nu,1}\Gamma^{12} + e^{-2\lambda_{2}}F_{\mu\nu,2}\Gamma^{34})\right]\epsilon$$
(90)

$$\delta\lambda^{(1)} = \left[\frac{m}{4}(e^{2\lambda_1} - e^{-4(\lambda_1 + \lambda_2)}) - \frac{1}{4}\gamma^{\mu}\partial_{\mu}(3\lambda_1 + 2\lambda_2) - \frac{1}{8}\gamma^{\mu\nu}e^{-2\lambda_1}F_{\mu\nu,1}\Gamma^{12}\right]\epsilon \tag{91}$$

$$\delta\lambda^{(2)} = \left[\frac{m}{4}(e^{2\lambda_2} - e^{-4(\lambda_1 + \lambda_2)}) - \frac{1}{4}\gamma^{\mu}\partial_{\mu}(2\lambda_1 + 3\lambda_2) - \frac{1}{8}\gamma^{\mu\nu}e^{-2\lambda_2}F_{\mu\nu,2}\Gamma^{34}\right]\epsilon \tag{92}$$

Where the spin connection components, for the metric considered in the text (22) are given by

$$\omega_{\hat{u}}^{u,r} = f' \qquad \omega_{\hat{x}}^{xr} = \omega_{\hat{y}}^{yr} = \frac{e^{g-f}g'}{y} \qquad \omega_{\hat{x}}^{xy} = -\frac{1}{y}$$
 (93)

Here the hatted indices are curved while the unhatted one are flat.

The vanishing gravitino and gaugino equations read

$$\partial_{\hat{u}}\epsilon + \frac{\omega_{\hat{u}}^{u,r}\gamma_{ur}}{2}\epsilon + \frac{m}{4}e^{-4(\lambda_1 + \lambda_2) + f}\gamma_u\epsilon + \frac{\lambda_1' + \lambda_2'}{2}\gamma_{ur}\epsilon = 0$$
 (94)

$$\left[\partial_{\hat{x}}\epsilon + \frac{1}{2}\omega_{\hat{x}}^{xy}\gamma_{xy}\epsilon + \frac{k}{2}(A_{\hat{x}}^{(1)}\Gamma^{12} + A_{\hat{x}}^{(2)}\Gamma^{34})\epsilon + \frac{1}{2}\omega_{\hat{x}}^{xr}\gamma_{xr}\epsilon + \frac{m}{4y}e^{-4(\lambda_{1}+\lambda_{2})+g}\gamma_{x\epsilon} + \frac{e^{g-f}(\lambda_{1}' + \lambda_{2}')}{2u}\gamma_{xr} + \frac{ye^{-g}}{2}(e^{-2\lambda_{1}}F_{xy}^{(1)} + e^{-2\lambda_{2}}F_{xy}^{(2)})\gamma_{y}\epsilon = 0$$

$$(95)$$

$$m(e^{2\lambda_1} - e^{-4(\lambda_1 + \lambda_2)})\epsilon - e^{-f}(3\lambda_1' + 2\lambda_2')\gamma_r\epsilon - e^{-2g - 2\lambda_1}y^2 F_{xy}^{(1)}\gamma_{xy}\Gamma^{(12)}\epsilon = 0$$
 (96)

$$m(e^{2\lambda_1} - e^{-4(\lambda_1 + \lambda_2)})\epsilon - e^{-f}(2\lambda_1' + 3\lambda_2')\gamma_r\epsilon - e^{-2g - 2\lambda_2}y^2 F_{xy}^{(2)}\gamma_{xy}\Gamma^{(34)}\epsilon = 0$$
 (97)

$$\partial_{\hat{r}}\epsilon + \frac{m}{4}e^{-4(\lambda_1 + \lambda_2) + f}\gamma_r\epsilon + \frac{\lambda_1' + \lambda_2'}{2}\epsilon = 0$$
(98)

Imposing the conditions on the spinor

$$\gamma_r \epsilon = \epsilon, \quad \gamma_{xy} \epsilon = i\epsilon, \quad \Gamma^{(12)} \epsilon = i\epsilon, \quad \Gamma^{(34)} \epsilon = i\epsilon, \quad \partial_{u,y,t,z,x,w} \epsilon = 0,$$
 (99)

Different choices of signs in (99) are related by simple parity transformations. we choose  $\eta = \beta = \alpha = \gamma = 1$  and evaluating over the configurations  $A_x^{(1)} = \frac{a}{y} A_x^{(2)} = \frac{b}{y}$ 

$$f' = -e^{f-4(\lambda_1 + \lambda_2)} - \lambda_1' - \lambda_2'$$

$$g' = -e^f \left[ \frac{1}{2} e^{-4(\lambda_1 + \lambda_2)} - \kappa e^{-2g} (ae^{-2\lambda_1} + be^{-2\lambda_2}) \right] - (\lambda_1' + \lambda_2')$$

$$3\lambda_1' + 2\lambda_2' = e^f \left[ 2(e^{2\lambda_1} - e^{-4(\lambda_1 + \lambda_2)}) + \kappa ae^{-2g-2\lambda_1} \right]$$

$$3\lambda_2' + 2\lambda_1' = e^f \left[ 2(e^{2\lambda_2} - e^{-4(\lambda_1 + \lambda_2)}) + \kappa be^{-2g-2\lambda_2} \right]$$

$$1 = 4(a+b)$$
(100)

where  $\kappa = 1$  for  $H^2$  and  $\kappa = -1$  for  $S^2$ . In the  $\mathcal{N} = 2$  configurations we set a = 1/4, b = 0,  $3\lambda_2 + 2\lambda_1 = 0$  and we define  $\lambda = \lambda_2$ . Then we get the equations (23). For  $\mathcal{N} = 1$  configurations we choose a = b = 1/8 and  $\lambda_1 = \lambda_2 = -\phi/2$  we get the equations (27).

We can also calculate the supergravity "central charge" of the four dimensional conformal field theory, which is given by

$$c = \frac{\pi R_{AdS_5}^3}{8G_N^5} \tag{101}$$

in a normalization where  $c = N^2/4$  for  $\mathcal{N} = 4$  U(N) SYM. In our case, a calculation parallel to that done for the D3 brane gives

$$c = \frac{8}{3}(g-1)e^{2g+3f_0}N^3 \tag{102}$$

where  $f_0$  is the constant piece in f defined as  $f = f_0 - \log r$  where f, r are those of the ansatz (22). From (25) and (28) we can see that we get  $e^{2g+3f_0} = 1/8$  for the  $\mathcal{N} = 2$  case and  $e^{2g+3f_0} = 27/2^8$  for the  $\mathcal{N} = 1$  case. In the  $\mathcal{N} = 2$  case we can calculate the effective number of vector multiplets as follows. We know that the conformal anomaly coefficients a, c (see [69, 70]) are equal to leading order in N for theories that have a gravity dual. This implies that this theory has the same number of vector and hypermultiplets. Then from the formulas for a, c in [71] and (101) we find  $n_V = n_H = \frac{4}{3}(g-1)N^3$ .

Let us finish this appendix by writing the expression for the warp factor that is obtained when uplifting this solutions to M theory [33]

$$g_{00}^{(11)} = W g_{00}^{(7)}$$
,  $W^3 = e^{-4(\lambda_1 + \lambda_2)} \mu_0^2 + e^{2\lambda_2} \mu_1^2 + e^{2\lambda_1} \mu_2^2$  (103)

where  $\mu_i$  parametrize  $S^2$ ,  $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$ .

#### 7.4 D4 brane solutions

Here we consider the solutions that we get when we compactify a direction along the M5 worldvolume on  $S^1$  and we reduce the 11 dimensional solution to a 10 dimensional solution describing a D4 brane wrapped on the Riemann surface.

We find that the dilaton and ten dimensional metric are

$$e^{2\phi} = We^{3f}$$

$$ds_{str}^{2} = e^{2\phi/3} ds_{11}^{2} \Big|_{10}$$

$$W^{2} = e^{2\lambda} \cos^{2} \theta + e^{-3\lambda} \sin^{2} \theta$$
(104)

where the right hand side of the second line denotes the eleven dimensional metric along the ten directions of the type IIA solution, the factor of the dilaton produces the string metric in ten dimensions. The functions  $\lambda$ , f that appear here are those of the solution (24). In principle we could also find the various components of the four form field strength of type IIA from that of the 11 dimensional solution. Of course we can do the same with the other solution we found, the one with  $\mathcal{N}=1$  supersymmetry in four dimensions.

#### References

- [1] M. Alishahiha and Y. Oz, "AdS/CFT and BPS strings in four dimensions," *Phys. Lett.* **B465** (1999) 136, hep-th/9907206.
- [2] M. Cvetic, H. Lu, C. N. Pope, and J. F. Vazquez-Poritz, "AdS in warped spacetimes," hep-th/0005246.
- [3] A. Fayyazuddin and D. J. Smith, "Localized intersections of M5-branes and four-dimensional superconformal field theories," *JHEP* **04** (1999) 030, hep-th/9902210.
- [4] M. Bershadsky, C. Vafa, and V. Sadov, "D-Branes and Topological Field Theories," Nucl. Phys. B463 (1996) 420-434, hep-th/9511222.
- [5] D. Z. Freedman and B. Zwiebach, "Magnetovacs in gauged N=4 supergravity," Nucl. Phys. B237 (1984) 573.
- [6] A. H. Chamseddine and W. A. Sabra, "Magnetic strings in five dimensional gauged supergravity theories," Phys. Lett. B477 (2000) 329, hep-th/9911195.
- [7] D. Klemm and W. A. Sabra, "Supersymmetry of black strings in D = 5 gauged supergravities," Phys. Rev. D62 (2000) 024003, hep-th/0001131.
- [8] S. Cacciatori, D. Klemm, W. A. Sabra, and D. Zanon, "Entropy of black holes in D = 5, N = 2 supergravity and AdS central charges," hep-th/0004077.
- [9] A. H. Chamseddine and W. A. Sabra, "Magnetic and dyonic black holes in D = 4 gauged supergravity," hep-th/0003213.
- [10] L. Randall and R. Sundrum, "An alternative to compactification," *Phys. Rev. Lett.* **83** (1999) 4690, hep-th/9906064.
- [11] R. Kallosh and A. Linde, "Supersymmetry and the brane world," JHEP 02 (2000) 005, hep-th/0001071.
- [12] M. Wijnholt and S. Zhukov, "On the uniqueness of black hole attractors," hep-th/9912002.
- [13] G. W. Gibbons and N. D. Lambert, "Domain walls and solitons in odd dimensions," hep-th/0003197.
- [14] G. Papadopoulos, J. G. Russo, and A. A. Tseytlin, "Curved branes from string dualities," Class. Quant. Grav. 17 (2000) 1713, hep-th/9911253.
- [15] H. song Yang, "Localized intersecting brane solutions of D = 11 supergravity," hep-th/9902128.
- [16] Y. Kiem and D. Park, "BTZ black holes from the five-dimensional general relativity with a negative cosmological constant," Phys. Lett. B450 (1999) 41, hep-th/9810205.
- [17] D. Youm, "Localized intersecting BPS branes," hep-th/9902208.
- [18] A. Loewy, "Semi localized brane intersections in SUGRA," Phys. Lett. B463 (1999) 41, hep-th/9903038.
- [19] A. Gomberoff, D. Kastor, D. Marolf, and J. Traschen, "Fully localized brane intersections: The plot thickens," *Phys. Rev.* **D61** (2000) 024012, hep-th/9905094.

- [20] D. Youm, "Supergravity solutions for BI dyons," Phys. Rev. D60 (1999) 105006, hep-th/9905155.
- [21] S. A. Cherkis, "Supergravity solution for M5-brane intersection," hep-th/9906203.
- [22] A. Fayyazuddin and D. J. Smith, "Warped AdS near-horizon geometry of completely localized intersections of M5-branes," hep-th/0006060.
- [23] J. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. 2 (1998) 231–252, hep-th/9711200.
- [24] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, "Supergravity and the large N limit of theories with sixteen supercharges," Phys. Rev. D58 (1998) 046004, hep-th/9802042.
- [25] C. Vafa and E. Witten, "A Strong coupling test of S duality," Nucl. Phys. B431 (1994) 3-77, hep-th/9408074.
- [26] M. Bershadsky, A. Johansen, V. Sadov, and C. Vafa, "Topological reduction of 4-d SYM to 2-d sigma models," Nucl. Phys. B448 (1995) 166–186, hep-th/9501096.
- [27] S. S. Gubser, "Curvature singularities: The good, the bad, and the naked," hep-th/0002160.
- [28] Y. Oz, "Warped compactifications and AdS/CFT," hep-th/0004009.
- [29] J. Polchinski and M. J. Strassler, "The string dual of a confining four-dimensional gauge theory," hep-th/0003136.
- [30] N. Seiberg and E. Witten, "The D1/D5 system and singular CFT," JHEP 04 (1999) 017, hep-th/9903224.
- [31] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. 2 (1998) 253–291, hep-th/9802150.
- [32] M. Gunaydin, L. J. Romans, and N. P. Warner, "Gauged N=8 supergravity in five dimensions," Phys. Lett. B154 (1985) 268.
- [33] M. Cvetic et. al., "Embedding AdS black holes in ten and eleven dimensions," Nucl. Phys. B558 (1999) 96, hep-th/9903214.
- [34] M. Gunaydin, G. Sierra, and P. K. Townsend, "Gauging the d=5 Maxwell-Einstein supergravity theories: more on Jordan algebras," Nucl. Phys. B253 (1985) 573.
- [35] V. Balasubramanian, P. Kraus, and A. Lawrence, "Bulk vs. boundary dynamics in anti-de Sitter spacetime," *Phys. Rev.* **D59** (1999) 046003, hep-th/9805171.
- [36] D. Z. Freedman, S. S. Gubser, K. Pilch, and N. P. Warner, "Continuous distributions of D3-branes and gauged supergravity," hep-th/9906194.
- [37] E. Witten, "On the conformal field theory of the Higgs branch," JHEP 07 (1997) 003, hep-th/9707093.
- [38] M. Berkooz and H. Verlinde, "Matrix theory, AdS/CFT and Higgs-Coulomb equivalence," *JHEP* 11 (1999) 037, hep-th/9907100.
- [39] O. Aharony and M. Berkooz, "IR dynamics of d = 2, N = (4,4) gauge theories and DLCQ of 'little string theories'," *JHEP* **10** (1999) 030, hep-th/9909101.
- [40] D. Marolf and A. Peet, "Brane baldness vs. superselection sectors," Phys. Rev. D60 (1999) 105007, hep-th/9903213.
- [41] S. Katz, D. R. Morrison, and M. R. Plesser, "Enhanced Gauge Symmetry in Type II String Theory," Nucl. Phys. **B477** (1996) 105–140, hep-th/9601108.
- [42] J. Curtis G. Callan and J. M. Maldacena, "Brane dynamics from the Born-Infeld action," Nucl. Phys. B513 (1998) 198, hep-th/9708147.
- [43] A. Imaanpur, "N = 4 SYM on Sigma x S(2) and its topological reduction," Nucl. Phys. **B551** (1999) 467, hep-th/9803042.

- [44] p. c. C. Vafa.
- [45] J. T. Liu and R. Minasian, "Black holes and membranes in AdS(7)," Phys. Lett. B457 (1999) 39, hep-th/9903269.
- [46] O. Aharony, M. Berkooz, S. Kachru, N. Seiberg, and E. Silverstein, "Matrix description of interacting theories in six dimensions," Adv. Theor. Math. Phys. 1 (1998) 148–157, hep-th/9707079.
- [47] O. Aharony, M. Berkooz, and N. Seiberg, "Light-cone description of (2,0) superconformal theories in six dimensions," Adv. Theor. Math. Phys. 2 (1998) 119, hep-th/9712117.
- [48] A. Kapustin and S. Sethi, "The Higgs branch of impurity theories," Adv. Theor. Math. Phys. 2 (1998) 571, hep-th/9804027.
- [49] N. Seiberg and E. Witten, "Gauge dynamics and compactification to three dimensions," hep-th/9607163.
- [50] K. Intriligator and N. Seiberg, "Mirror symmetry in three dimensional gauge theories," Phys. Lett. B387 (1996) 513-519, hep-th/9607207.
- [51] A. Hanany and E. Witten, "Type IIB superstrings, BPS monopoles, and three-dimensional gauge dynamics," Nucl. Phys. B492 (1997) 152–190, hep-th/9611230.
- [52] C. V. Johnson, A. W. Peet, and J. Polchinski, "Gauge theory and the excision of repulson singularities," Phys. Rev. D61 (2000) 086001, hep-th/9911161.
- [53] S. Kachru and E. Silverstein, "4d conformal theories and strings on orbifolds," Phys. Rev. Lett. 80 (1998) 4855, hep-th/9802183.
- [54] D. Z. Freedman, S. S. Gubser, K. Pilch, and N. P. Warner, "Renormalization group flows from holography supersymmetry and a c-theorem," hep-th/9904017.
- [55] M. Cvetic, H. Lu, and C. N. Pope, "Domain walls and massive gauged supergravity potentials," hep-th/0001002.
- [56] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and R. Sundrum, "A small cosmological constant from a large extra dimension," Phys. Lett. B480 (2000) 193, hep-th/0001197.
- [57] A. Strominger, "Superstrings with torsion," Nucl. Phys. B274 (1986) 253.
- [58] E. Witten, "Strong Coupling Expansion Of Calabi-Yau Compactification," Nucl. Phys. B471 (1996) 135–158, hep-th/9602070.
- [59] K. Dasgupta, G. Rajesh, and S. Sethi, "M theory, orientifolds and G-flux," JHEP 08 (1999) 023, hep-th/9908088.
- [60] C. S. Chan, P. L. Paul, and H. Verlinde, "A note on warped string compactification," hep-th/0003236.
- [61] K. Behrndt and S. Gukov, "Domain walls and superpotentials from M theory on Calabi- Yau three-folds," Nucl. Phys. **B580** (2000) 225, hep-th/0001082.
- [62] P. Mayr, "Stringy world branes and exponential hierarchies," hep-th/0006204.
- [63] P. Horava and E. Witten, "Heterotic and type I string dynamics from eleven dimensions," Nucl. Phys. B460 (1996) 506–524, hep-th/9510209.
- [64] M. Cvetic and H. H. Soleng, "Supergravity domain walls," Phys. Rept. 282 (1997) 159, hep-th/9604090.
- [65] O. DeWolfe, D. Z. Freedman, S. S. Gubser, and A. Karch, "Modeling the fifth dimension with scalars and gravity," hep-th/9909134.
- [66] L. J. Romans, "Massive N=2a supergravity in ten-dimensions," Phys. Lett. **B169** (1986) 374.
- [67] J. Polchinski and E. Witten, "Evidence for Heterotic Type I String Duality," Nucl. Phys. B460 (1996) 525–540, hep-th/9510169.

- [68] A. Chamblin, R. Emparan, C. V. Johnson, and R. C. Myers, "Charged AdS black holes and catastrophic holography," *Phys. Rev.* **D60** (1999) 064018, hep-th/9902170.
- [69] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, "Large N field theories, string theory and gravity," *Phys. Rept.* **323** (2000) 183, hep-th/9905111.
- [70] M. Henningson and K. Skenderis, "The holographic Weyl anomaly," JHEP 07 (1998) 023, hep-th/9806087.
- [71] M. J. Duff, "Twenty years of the Weyl anomaly," Class. Quant. Grav. 11 (1994) 1387–1404, hep-th/9308075.